Lecture 2

2.1. Introduction to Probability

Summary

In this lecture we introduced basic probability concepts and illustrated how probability analysis can be used to provide helpful information for decision making. We described how probability can be interpreted as a numerical measure of the likelihood that an event will occur. In addition, we saw that the probability of an event can be computed either by summing the probabilities of the experimental outcomes (sample points) comprising the event or by using the relationships established by the addition, conditional probability, and multiplication laws of probability. For cases in which additional information is available, we showed how Bayes' theorem can be used to obtain revised or posterior probabilities.

Glossary

Probability A numerical measure of the likelihood that an event will occur.

Experiment A process that generates well-defined outcomes.

Sample space The set of all experimental outcomes.

Sample point An element of the sample space. A sample point represents an experimental outcome.

Tree diagram A graphical representation that helps in visualizing a multiple-step experiment.

Basic requirements for assigning probabilities Two requirements that restrict the manner in which probability assignments can be made: (1) for each experimental outcome E_i we must have $0 \le P(E) \le 1$; (2) considering all experimental outcomes, we must have $P(E_1) + P(E_2) + \dots + P(E_n) = 1.0$.

Classical method A method of assigning probabilities that is appropriate when all the experimental outcomes are equally likely.

Relative frequency method A method of assigning probabilities that is appropriate when data are available to estimate the proportion of the time the experimental outcome will occur if the experiment is repeated a large number of times.

Subjective method A method of assigning probabilities on the basis of judgment.

Event A collection of sample points.

Complement of A The event consisting of all sample points that are not in A.

Venn diagram A graphical representation for showing symbolically the sample space and operations involving events in which the sample space is represented by a rectangle and events are represented as circles within the sample space.

Union of *A* and *B* The event containing all sample points belonging to A or B or both. The union is denoted $A \cup B$.

Intersection of *A* and *B* The event containing the sample points belonging to both A and B. The intersection is denoted $A \cap B$.

Addition law A probability law used to compute the probability of the union of two events. It is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. For mutually exclusive events, $P(A \cap B) = 0$; in this case the addition law reduces to $P(A \cup B) = P(A) + P(B)$.

Mutually exclusive events Events that have no sample points in common; that is, $A \cap B$ is empty and $P(A \cap B) = 0$.

Conditional probability The probability of an event given that another event already occurred. The conditional probability of A given B is $P(A \mid B) = P(A \cap B)/P(B)$.

Joint probability The probability of two events both occurring; that is, the probability of the intersection of two events.

Marginal probability The values in the margins of a joint probability table that provide the probabilities of each event separately.

Independent events Two events A and B where $P(A \mid B) = P(A)$ or $P(B \mid \mathrm{A}) = P(B)$; that is, the events have no influence on each other.

Multiplication law A probability law used to compute the probability of the intersection of two events. It is $P(A \cap B) = P(B)P(A|B)$ or $P(A \cap B) = P(A)P(B|\mathrm{A})$. For independent events it reduces to $P(A \cap B) = P(A)P(B)$.

Prior probabilities Initial estimates of the probabilities of events.

Posterior probabilities Revised probabilities of events based on additional information

Bayes' theorem A method used to compute posterior probabilities.

Key formulas

Counting Rule for Combinations. Each notation is pronounced as "N-capital choose n"

$$\binom{N}{n} = C_n^N = C_N^n = {}_NC_n = C(N,n) = \frac{N!}{n!(N-n)!} \tag{2.1}$$

Counting Rule for Permutations

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!} \tag{2.2}$$

Computing Probability Using the Complement

$$P(A) = 1 - P(A^c) \tag{2.3}$$

Addition Law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{2.4}$$

Conditional Probability

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \tag{2.5}$$

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \tag{2.6}$$

Multiplication Law

$$P(A \cap B) = P(B)P(A \mid B) \tag{2.7}$$

$$P(A \cap B) = P(A)P(B \mid A) \tag{2.8}$$

Multiplication Law for Independent Events

$$P(A \cap B) = P(A)P(B) \tag{2.9}$$

Bayes' Theorem

$$P(A_i \mid B) = \frac{P(A_i)P(B \mid A_i)}{P(A_1)P(B \mid A_1) + P(A_2)P(B \mid A_2) + \cdots + P(A_n)P(B \mid A_n)} \tag{2.10}$$