CENTRE DE RECHERCHE PUBLIC

Microarray Center

the Luxembourg Bioinformatics Network

UNIVERSITÉ DU LUXEMBOURG

STATISTICAL DATA ANALYSIS IN EXCEL

Lecture 2

Interval Estimations for Means

dr. Petr Nazarov

petr.nazarov@crp-sante.lu

31-10-2011

- **Sampling distribution**
  - sample and population and their parameters
  - central limit theorem
  - types of sampling

- **Interval estimation**
  - interval estimation
  - population mean: σ known
  - population proportion
  - population mean: σ unknown
  - Student's distribution
  - estimation the size of a sample

SAMPLING DISTRIBUTION

Parameters

Population parameter
A numerical value used as a summary measure for a population (e.g., the mean μ, variance σ^2, standard deviation σ, *proportion* π)

POPULATION

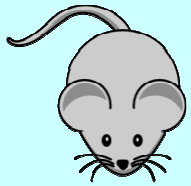
μ – mean
σ^2 – variance
N – number of elements (usually $N=\infty$)

SAMPLE

$m, \bar{x}$ – mean
s^2 – variance
n – number of elements

Sample statistic
A numerical value used as a summary measure for a sample (e.g., the sample mean m, the sample variance s^2, and the sample standard deviation s)

All existing laboratory
Mus musculus



mice.xls

790 mice from different strains
http://phenome.jax.org

ID	Strain	Sex	Starting age	Ending age	Starting weight	Ending weight	Weight change	Bleeding time	Ionized Ca in blood	Blood pH	Bone mineral density	Lean tissues weight	Fat weight
1	129S1/SvImJ	f	66	116	19.3	20.5	1.062	64	1.2	7.24	0.0605	14.5	4.4
2	129S1/SvImJ	f	66	116	19.1	20.8	1.089	78	1.15	7.27	0.0553	13.9	4.4
3	129S1/SvImJ	f	66	108	17.9	19.8	1.106	90	1.16	7.26	0.0546	13.8	2.9
368	129S1/SvImJ	f	72	114	18.3	21	1.148	65	1.26	7.22	0.0599	15.4	4.2
369	129S1/SvImJ	f	72	115	20.2	21.9	1.084	55	1.23	7.3	0.0623	15.6	4.3
370	129S1/SvImJ	f	72	116	18.8	22.1	1.176		1.21	7.28	0.0626	16.4	4.3
371	129S1/SvImJ	f	72	119	19.4	21.3	1.098	49	1.24	7.24	0.0632	16.6	5.4
372	129S1/SvImJ	f	72	122	18.3	20.1	1.098	73	1.17	7.19	0.0592	16	4.1
4	129S1/SvImJ	f	66	109	17.2	18.9	1.099	41	1.25	7.29	0.0513	14	3.2
5	129S1/SvImJ	f	66	112	19.7	21.3	1.081	129	1.14	7.22	0.0501	16.3	5.2
10	129S1/SvImJ	m	66	112	24.3	24.7	1.016	119	1.13	7.24	0.0533	17.6	6.8
364	129S1/SvImJ	m	72	114	25.3	27.2	1.075	64	1.25	7.27	0.0596	19.3	5.8
365	129S1/SvImJ	m	72	115	21.4	23.9	1.117	48	1.25	7.28	0.0563	17.4	5.7
366	129S1/SvImJ	m	72	118	24.5	26.3	1.073	59	1.25	7.26	0.0609	17.8	7.1
367	129S1/SvImJ	m	72	122	24	26	1.083	69	1.29	7.26	0.0584	19.2	4.6
6	129S1/SvImJ	m	66	116	21.6	23.3	1.079	78	1.15	7.27	0.0497	17.2	5.7
7	129S1/SvImJ	m	66	107	22.7	26.5	1.167	90	1.18	7.28	0.0493	18.7	7
8	129S1/SvImJ	m	66	108	25.4	27.4	1.079	35	1.24	7.26	0.0538	18.9	7.1
9	129S1/SvImJ	m	66	109	24.4	27.5	1.127	43	1.29	7.29	0.0539	19.5	7.1

SAMPLING DISTRIBUTION

Example: Making a Random Sampling

mice.xls

790 mice from different strains

http://phenome.jax.org

ID	Strain	Sex	Starting age	Ending age	Starting weight	Ending weight	Weight change	Bleeding time	Ionized Ca in blood	Blood pH	Bone mineral density	Lean tissues weight	Fat weight
1	129S1/SvImJ	f	66	116	19.3	20.5	1.062	64	1.2	7.24	0.0605	14.5	4.4
2	129S1/SvImJ	f	66	116	19.1	20.8	1.089	78	1.15	7.27	0.0553	13.9	4.4
3	129S1/SvImJ	f	66	108	17.9	19.8	1.106	90	1.16	7.26	0.0546	13.8	2.9
368	129S1/SvImJ	f	72	114	18.3	21	1.148	65	1.26	7.22	0.0599	15.4	4.2
369	129S1/SvImJ	f	72	115	20.2	21.9	1.084	55	1.23	7.3	0.0623	15.6	4.3
370	129S1/SvImJ	f	72	116	18.8	22.1	1.176		1.21	7.28	0.0626	16.4	4.3
371	129S1/SvImJ	f	72	119	19.4	21.3	1.098	49	1.24	7.24	0.0632	16.6	5.4
372	129S1/SvImJ	f	72	122	18.3	20.1	1.098	73	1.17	7.19	0.0592	16	4.1
4	129S1/SvImJ	f	66	109	17.2	18.9	1.099	41	1.25	7.29	0.0513	14	3.2
5	129S1/SvImJ	f	66	112	19.7	21.3	1.081	129	1.14	7.22	0.0501	16.3	5.2
10	129S1/SvImJ	m	66	112	24.3	24.7	1.016	119	1.13	7.24	0.0533	17.6	6.8
364	129S1/SvImJ	m	72	114	25.3	27.2	1.075	64	1.25	7.27	0.0596	19.3	5.8
365	129S1/SvImJ	m	72	115	21.4	23.9	1.117	48	1.25	7.28	0.0563	17.4	5.7
366	129S1/SvImJ	m	72	118	24.5	26.3	1.073	59	1.25	7.26	0.0609	17.8	7.1
367	129S1/SvImJ	m	72	122	24	26	1.083	69	1.29	7.26	0.0584	19.2	4.6
6	129S1/SvImJ	m	66	116	21.6	23.3	1.079	78	1.15	7.27	0.0497	17.2	5.7
7	129S1/SvImJ	m	66	107	22.7	26.5	1.167	90	1.18	7.28	0.0493	18.7	7
8	129S1/SvImJ	m	66	108	25.4	27.4	1.079	35	1.24	7.26	0.0538	18.9	7.1
9	129S1/SvImJ	m	66	109	24.4	27.5	1.127	43	1.29	7.29	0.0539	19.5	7.1

1. Add a column to the table
2. Fill it with **=RAND()**
3. Sort all the table by this column

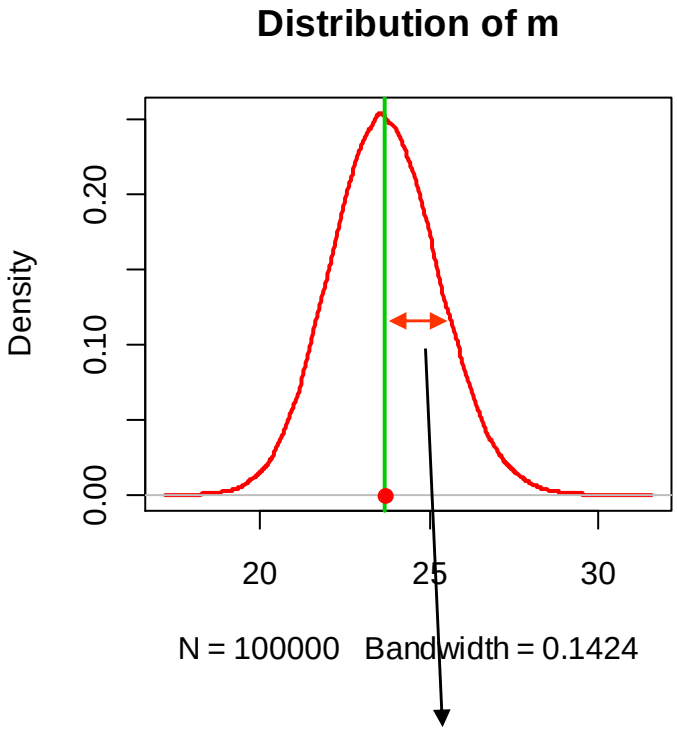
4. Assume that these mice is a population with size N=790. Build 3 samples with n=20

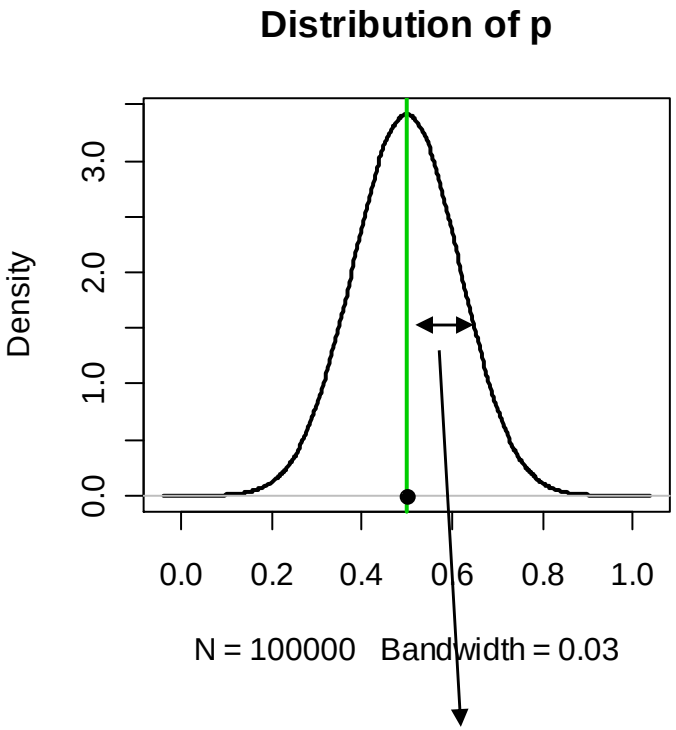
5. Calculate m, s for ending weight and p – proportion of males for each sample

Point estimator

The sample statistic, such as m, s, or p, that provides the point estimation the population parameters μ, σ, π.

Sampling distribution
A probability distribution consisting of all possible values of a sample statistic.





$$E(m) = \mu$$

$$E(p) = \pi$$

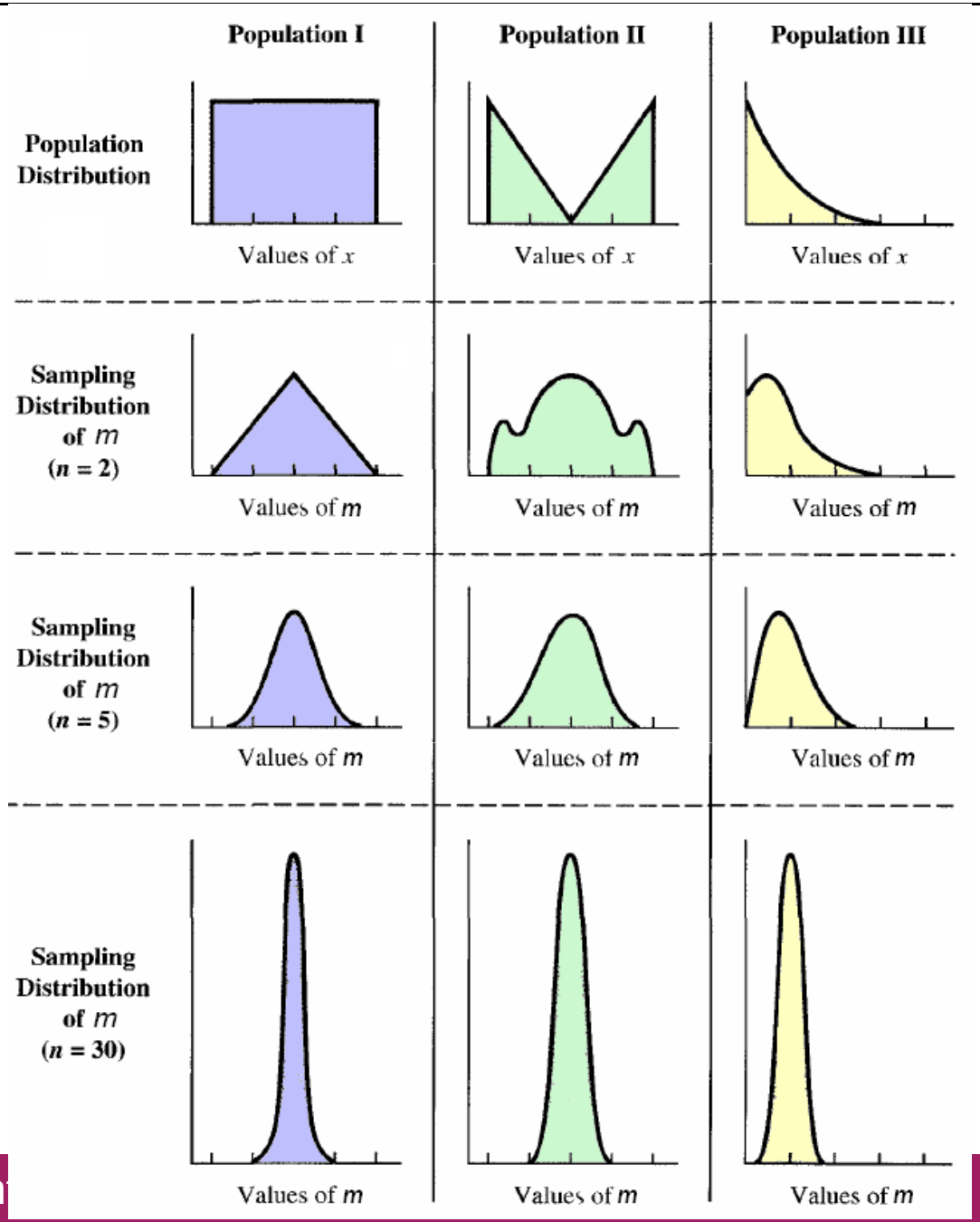
$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

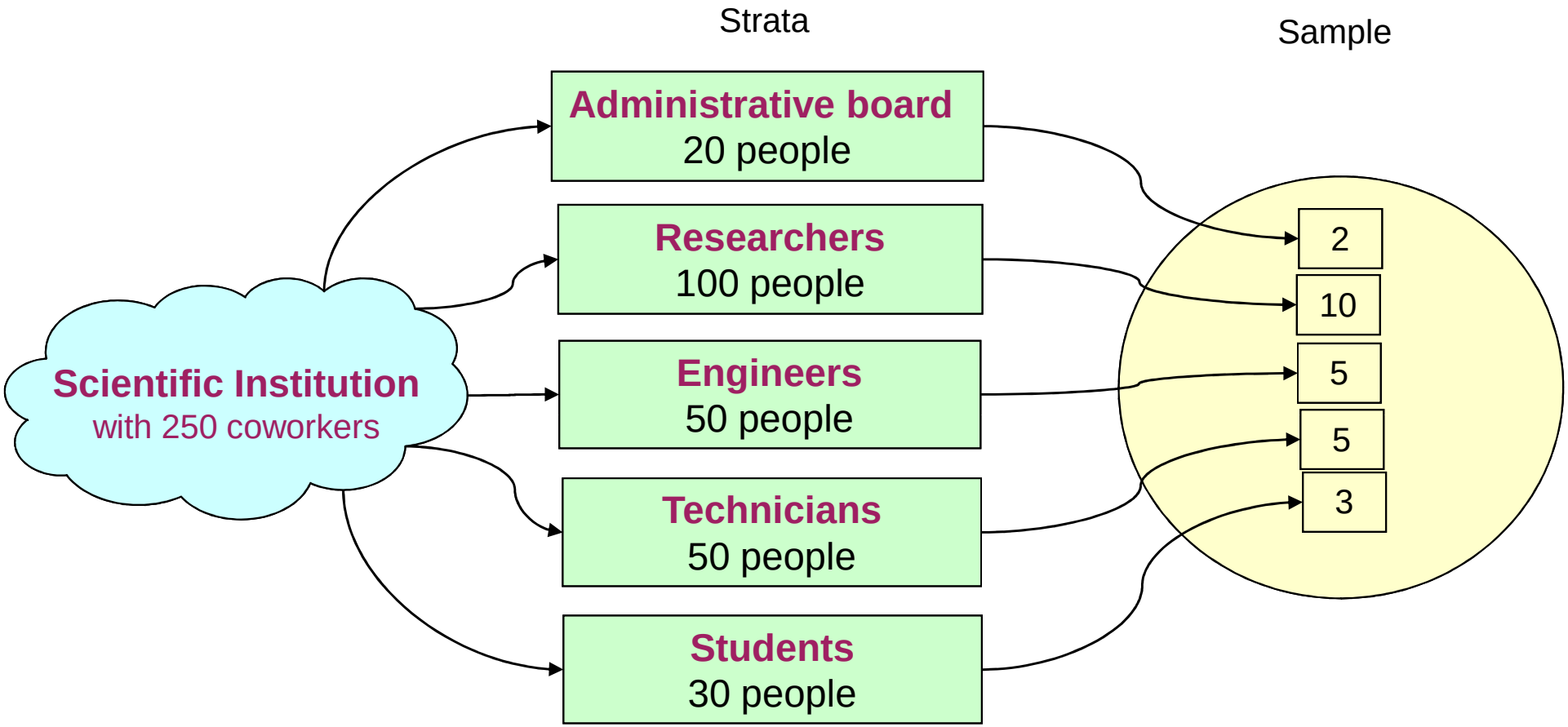
Central limit theorem

In selecting simple random sample of size *n* from a population, the ***sampling distribution of the sample mean m can be approximated by a normal distribution*** as the sample size becomes large

In practice if the sample size is ***n*>30**, the normal distribution is a good approximation for the sample mean for any initial distribution.

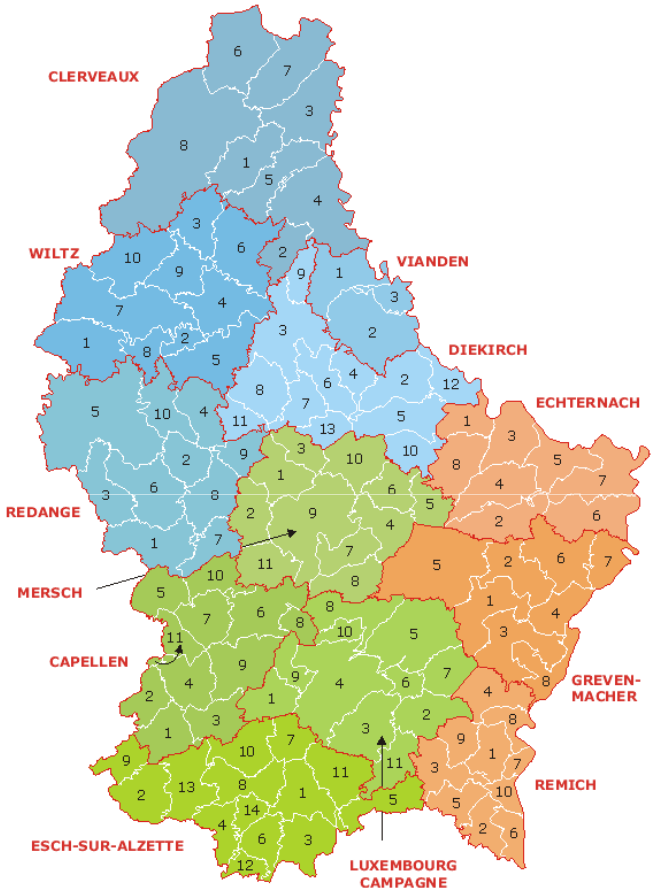


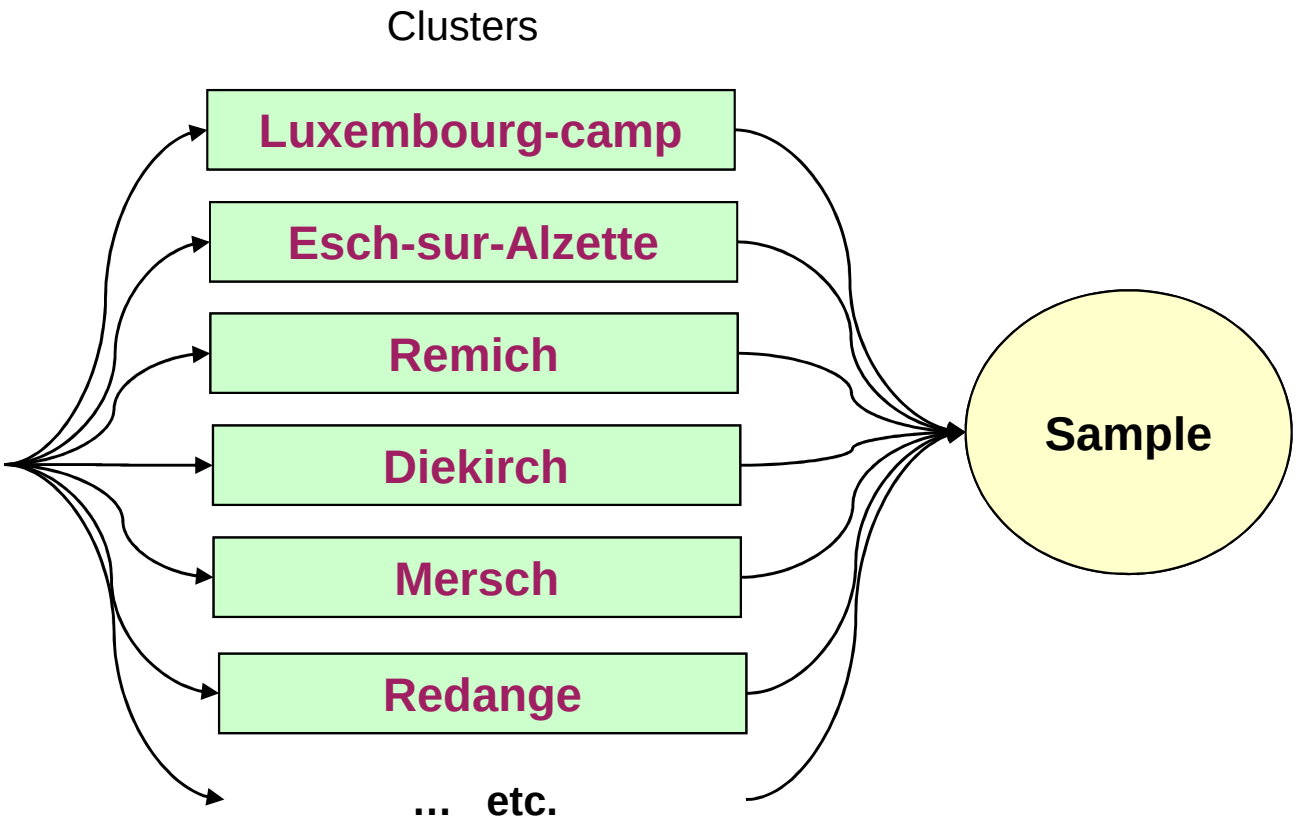
Stratified random sampling
A probability sampling method in which the population is first divided into strata and a simple random sample is then taken from each stratum.



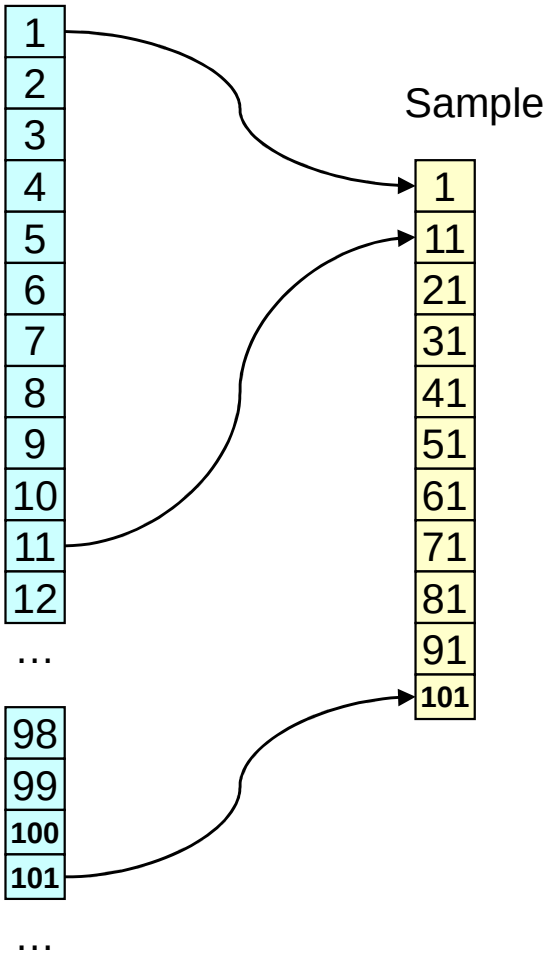
Cluster sampling

A probability sampling method in which the population is first divided into clusters and then a simple random sample of the clusters is taken.

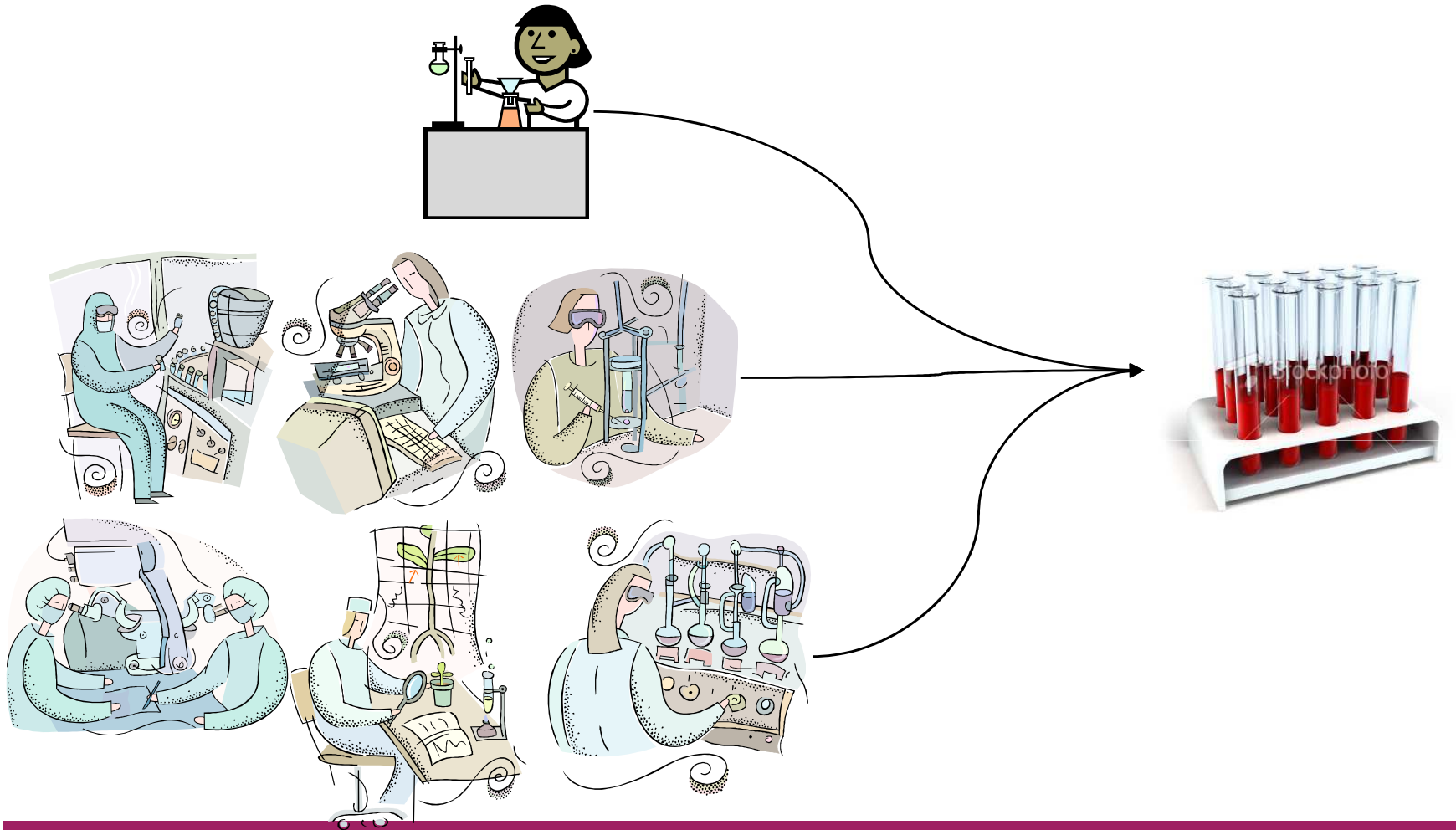




Systematic sampling
A probability sampling method in which we randomly select one of the first *k* elements and then select every *k*-th element thereafter.



Convenience sampling
A nonprobability method of sampling whereby elements are selected for the sample on the basis of convenience.

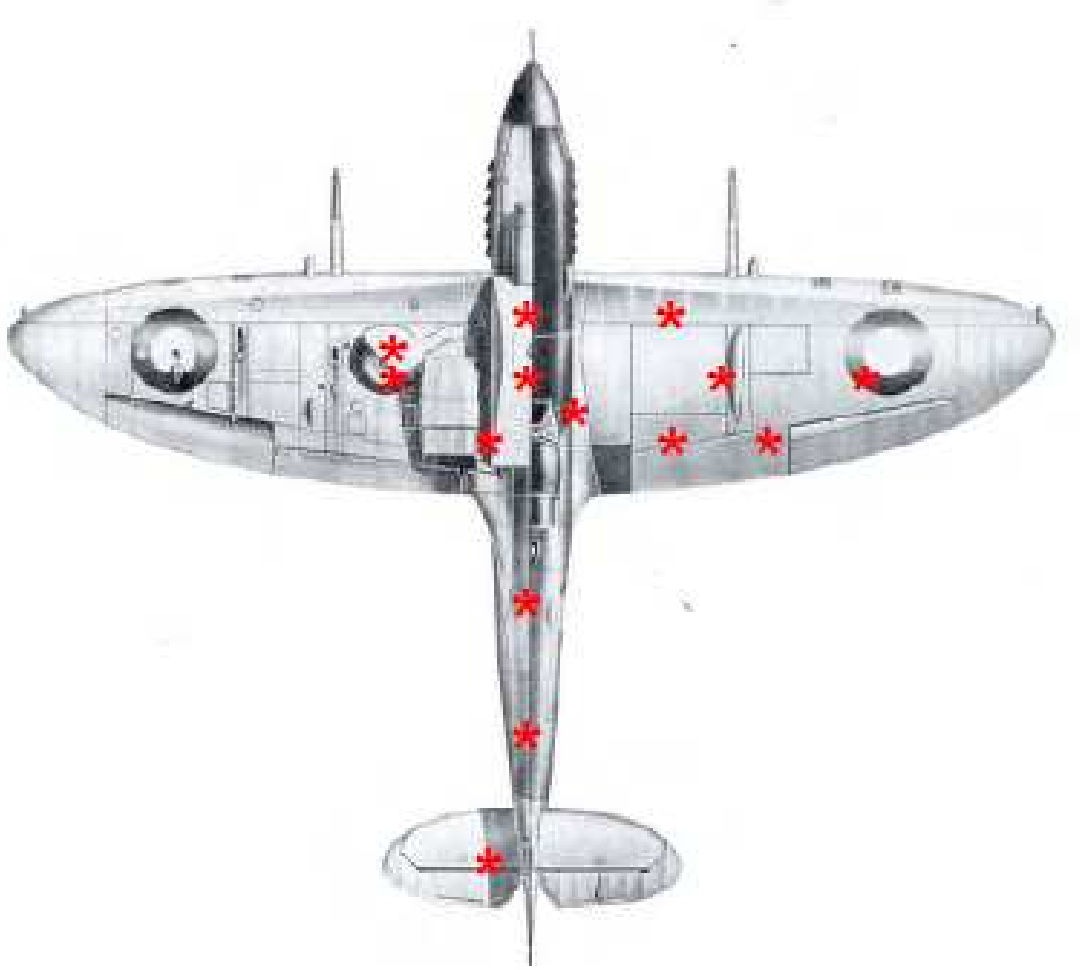


Judgment sampling
A nonprobability method of sampling whereby elements are selected for the sample based on the judgment of the person doing the study.

Perform of a selection of most confident or most experienced experts.

Were to put additional protection?

Spitfire: analysis of the damage



- **Interval estimation**
  - interval estimation
  - population mean: σ known
  - population proportion
  - population mean: σ unknown
  - Student's distribution
  - estimation the size of a sample

POPULATION AND SAMPLE

Parameters

Population parameter
A numerical value used as a summary measure for a population (e.g., the mean μ, variance σ^2, standard deviation σ, *proportion* π)

POPULATION

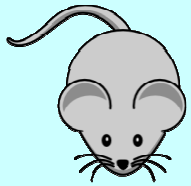
μ – mean
σ^2 – variance
N – number of elements (usually $N=\infty$)

SAMPLE

$m, \bar{x}$ – mean
s^2 – variance
n – number of elements

Sample statistic
A numerical value used as a summary measure for a sample (e.g., the sample mean m, the sample variance s^2, and the sample standard deviation s)

All existing laboratory
Mus musculus



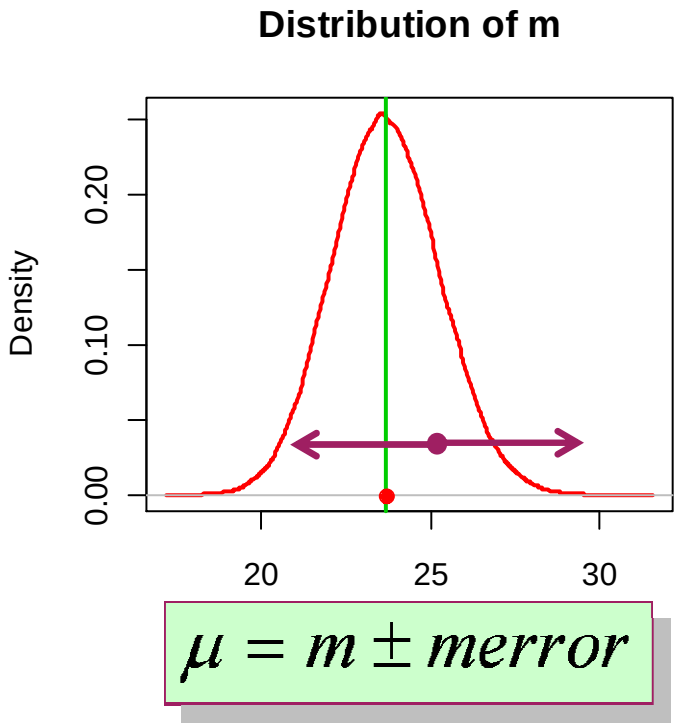
mice.txt

790 mice from different strains
http://phenome.jax.org

ID	Strain	Sex	Starting age	Ending age	Starting weight	Ending weight	Weight change	Bleeding time	Ionized Ca in blood	Blood pH	Bone mineral density	Lean tissues weight	Fat weight
1	129S1/SvImJ	f	66	116	19.3	20.5	1.062	64	1.2	7.24	0.0605	14.5	4.4
2	129S1/SvImJ	f	66	116	19.1	20.8	1.089	78	1.15	7.27	0.0553	13.9	4.4
3	129S1/SvImJ	f	66	108	17.9	19.8	1.106	90	1.16	7.26	0.0546	13.8	2.9
368	129S1/SvImJ	f	72	114	18.3	21	1.148	65	1.26	7.22	0.0599	15.4	4.2
369	129S1/SvImJ	f	72	115	20.2	21.9	1.084	55	1.23	7.3	0.0623	15.6	4.3
370	129S1/SvImJ	f	72	116	18.8	22.1	1.176		1.21	7.28	0.0626	16.4	4.3
371	129S1/SvImJ	f	72	119	19.4	21.3	1.098	49	1.24	7.24	0.0632	16.6	5.4
372	129S1/SvImJ	f	72	122	18.3	20.1	1.098	73	1.17	7.19	0.0592	16	4.1
4	129S1/SvImJ	f	66	109	17.2	18.9	1.099	41	1.25	7.29	0.0513	14	3.2
5	129S1/SvImJ	f	66	112	19.7	21.3	1.081	129	1.14	7.22	0.0501	16.3	5.2
10	129S1/SvImJ	m	66	112	24.3	24.7	1.016	119	1.13	7.24	0.0533	17.6	6.8
364	129S1/SvImJ	m	72	114	25.3	27.2	1.075	64	1.25	7.27	0.0596	19.3	5.8
365	129S1/SvImJ	m	72	115	21.4	23.9	1.117	48	1.25	7.28	0.0563	17.4	5.7
366	129S1/SvImJ	m	72	118	24.5	26.3	1.073	59	1.25	7.26	0.0609	17.8	7.1
367	129S1/SvImJ	m	72	122	24	26	1.083	69	1.29	7.26	0.0584	19.2	4.6
6	129S1/SvImJ	m	66	116	21.6	23.3	1.079	78	1.15	7.27	0.0497	17.2	5.7
7	129S1/SvImJ	m	66	107	22.7	26.5	1.167	90	1.18	7.28	0.0493	18.7	7
8	129S1/SvImJ	m	66	108	25.4	27.4	1.079	35	1.24	7.26	0.0538	18.9	7.1
9	129S1/SvImJ	m	66	109	24.4	27.5	1.127	43	1.29	7.29	0.0539	19.5	7.1

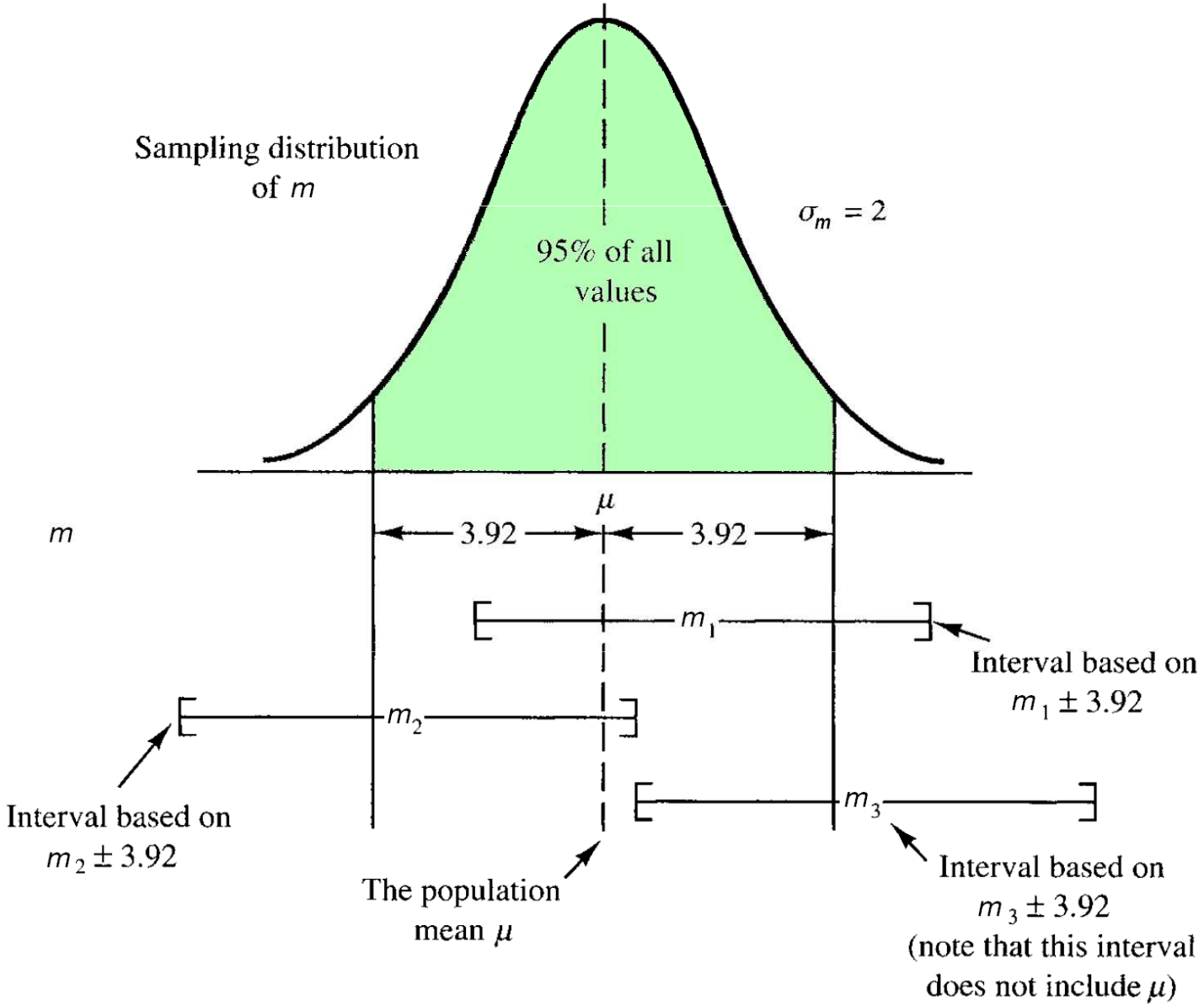
Interval estimate
An estimate of a population parameter that provides an interval believed to contain the value of the parameter. For the interval estimates in this chapter, it has the form: point estimate ± margin of error.

Margin of error
The ± value added to and subtracted from a point estimate in order to develop an interval estimate of a population parameter.



σ known
The condition existing when historical data or other information provides a good value for the population standard deviation prior to taking a sample. The interval estimation procedure uses this known value of o in computing the margin of error.

σ unknown
The condition existing when no good basis exists for estimating the population standard deviation prior to taking the sample. The interval estimation procedure uses the sample standard deviation s in computing the margin of error.


Sampling distribution of m
95% of all values
$\sigma_m = 2$
m
μ
3.92
3.92
m_1
Interval based on $m_1 \pm 3.92$
m_2
Interval based on $m_2 \pm 3.92$
m_3
The population mean μ
Interval based on $m_3 \pm 3.92$ (note that this interval does not include μ)

Confidence level

The confidence associated with an interval estimate. For example, if an interval estimation procedure provides intervals such that 95% of the intervals formed using the procedure will include the population parameter, the interval estimate is said to be constructed at the 95% confidence level.

Confidence interval

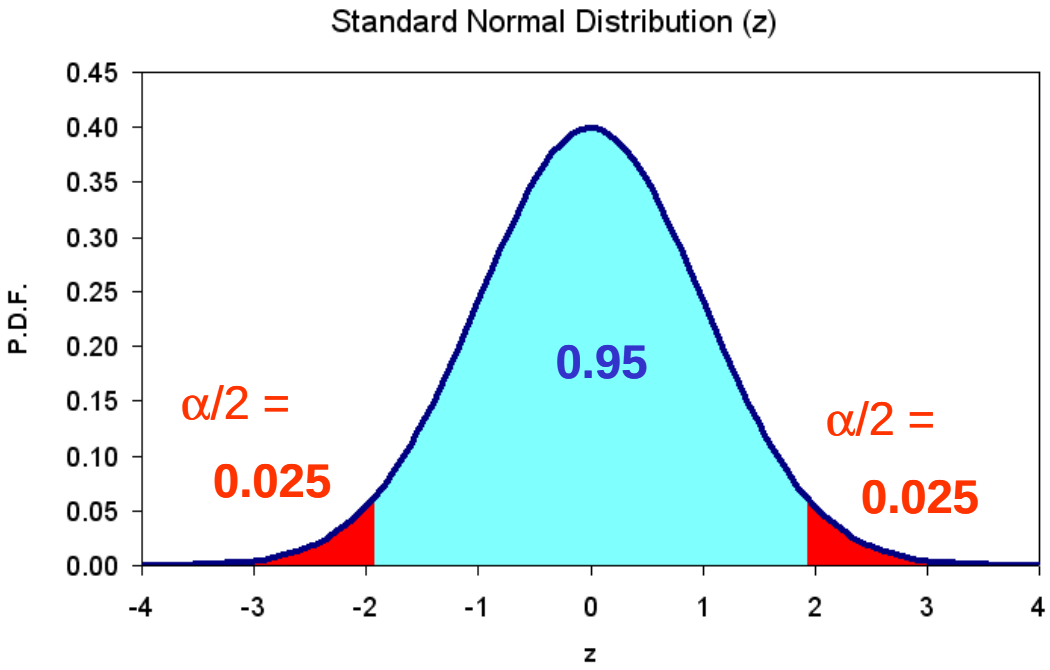
Another name for an interval estimate.

$$\mu = m \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

For 95 % confidence $\alpha = 0.05$, which means that in each tail we have 0.025. Corresponding $z_{\alpha/2} = 1.96$

In Excel use one of the following functions:

- `= CONFIDENCE(alpha, σ, n)`
- `= -NORMINV(alpha/2,0,1)*σ/SQRT(n)`

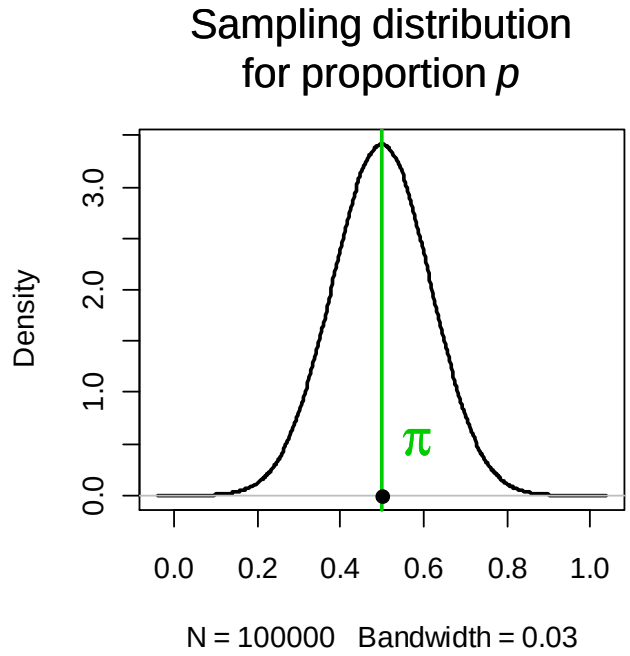


Population Proportion

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} \rightarrow \sigma_p = \sqrt{\frac{p(1-p)}{n}}$$

$$\pi = p \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

if $np \geq 5$ and $n(1-p) \geq 5$



Practical Work

pancreatitis.txt

Define a 95% confidence interval for **never-smoking** proportion of people coming to a hospital

n=	270
p(never)=	**0.214815**
sp=	0.024994
E=	**0.048988**

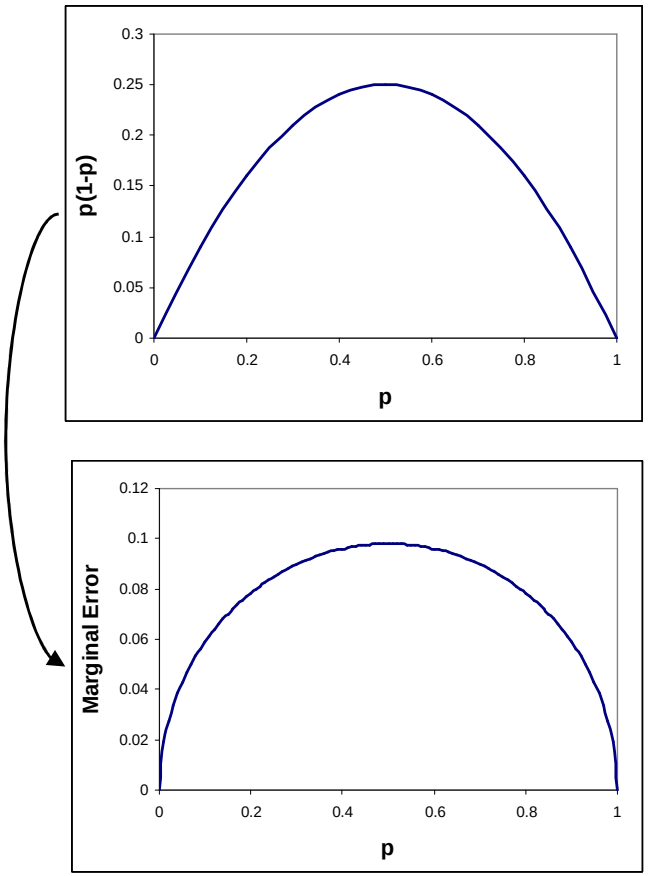
for 95% confidence $z_{0.025} = 1.96$

$\pi = 21.5 \pm 4.9$ %

Population Proportion: Some Practical Aspects

$$\pi = p \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

1. The normal distribution is applicable only when enough data points are observed. The rule of thumb is: $np \geq 5$ and $n(1-p) \geq 5$



2. The maximal marginal error is observed when p=0.5

3. The estimation of the sample size can be obtained:

$$n = \frac{z_{\alpha/2}^2 p(1-p)}{E^2}$$

$np \geq 5$ and $n(1-p) \geq 5$

where p is a best guess for π or the result of a preliminary study

Assume that we have a sample of 20 mice and would like to estimate an average size of a mice in population.

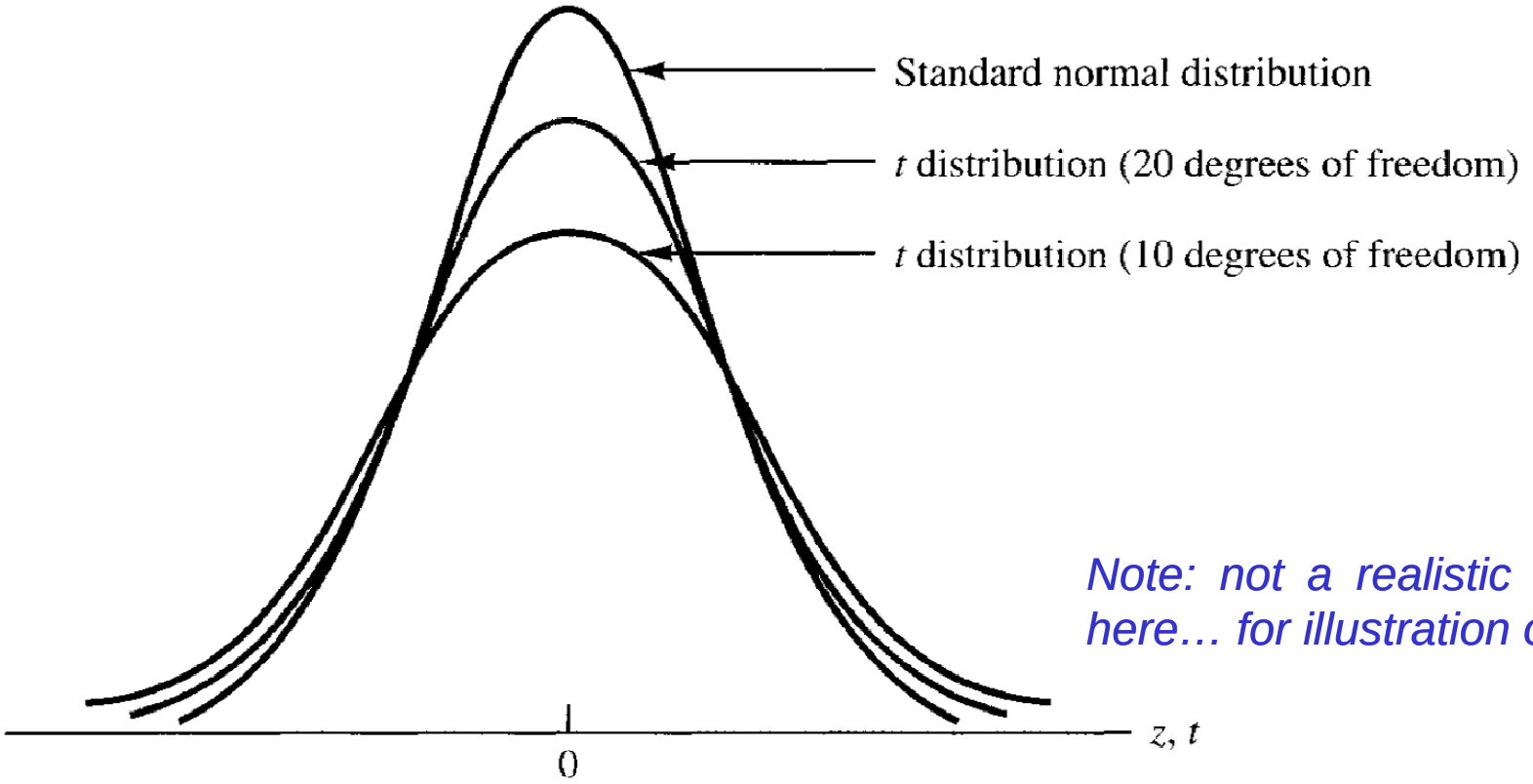
Weight
39.9
19.8
32.4
21
27.5
20.8
21.3
40
10.7
22.6
27
10.8
20.9
14.7
31.4
17.2
11.4
19.1
31.3
14.8

$m = 22.73$

$s = 8.84$

$$\sigma_m = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$$

As we replace σ → *s*, we introduce an additional error and this change the distribution from *z* to *t* (Student)



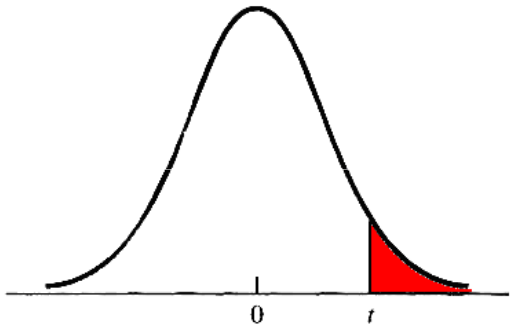
Note: not a realistic scale here… for illustration only

_t_-distribution

A family of probability distributions that can be used to develop an interval estimate of a population mean whenever the population standard deviation σ is unknown and is estimated by the sample standard deviation s.

Degrees of freedom

A parameter of the t-distribution. When the t distribution is used in the computation of an interval estimate of a population mean, the appropriate t distribution has $n - 1$ degrees of freedom, where n is the size of the simple random sample.



Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604

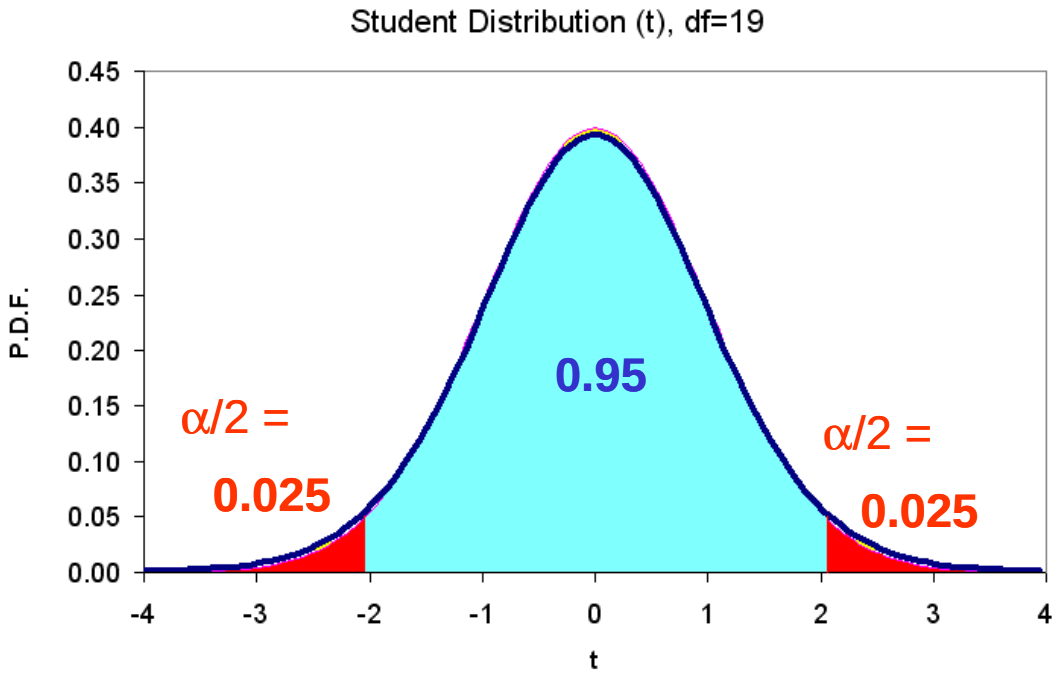
Population Mean: σ Unknown

Weight
39.9
19.8
32.4
21
27.5
20.8
21.3
40
10.7
22.6
27
10.8
20.9
14.7
31.4
17.2
11.4
19.1
31.3
14.8

m =	22.73
s =	8.84

s(m) =	1.98
t =	2.09
m.e. =	4.14

$$\mu = m \pm t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}}$$



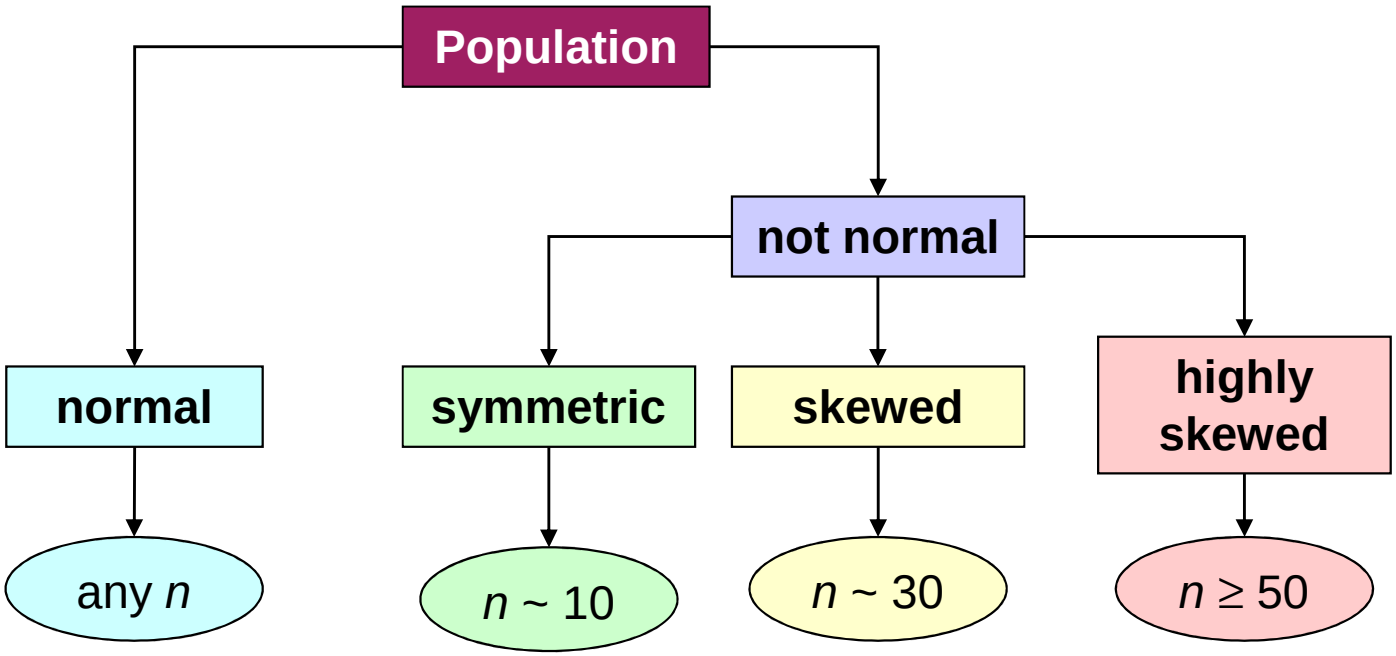
In Excel use:

- `= TINV(alpha, degree-of-freedom)` !!!

Population Mean: Practical Advices

$$\mu = m \pm t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}}$$

Advice 1



Advice 2

if n >100 you can use z-statistics instead of t-statistics (error will be <1.5%)

Let's focus on another aspect: how to select a proper number of experiments.

$$\mu = m \pm E(n,\sigma)$$

$$E(n,\sigma) = E$$

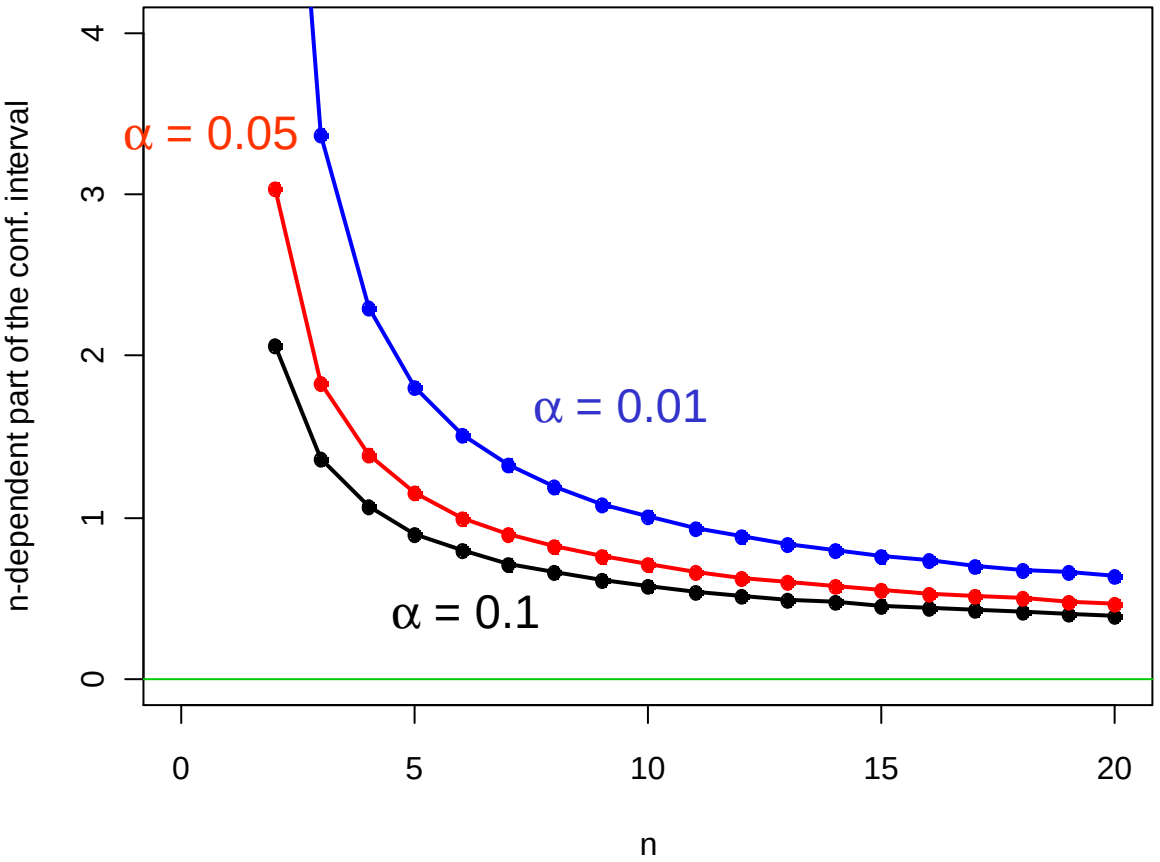
$$n - ?$$

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

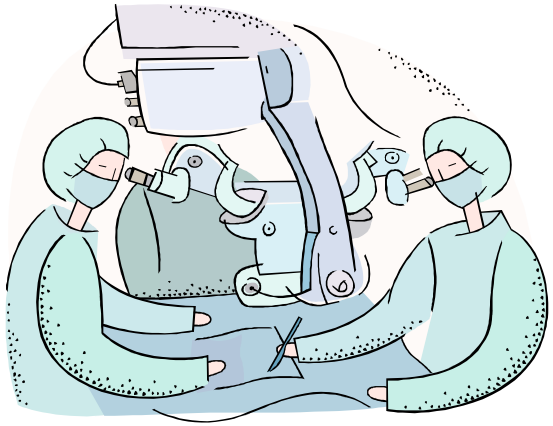
$$n = \frac{z_{\alpha/2}^2 \sigma^2}{E^2}$$

$$n = \frac{z_{\alpha/2}^2 \sigma^2}{E^2}$$

Effect of the Sample Size



Thank you for your attention



to be continued…