CENTRE DE RECHERCHE PUBLIC

Microarray Center

UNIVERSITÉ DU LUXEMBOURG

the Luxembourg Bioinformatics Network

STATISTICAL DATA ANALYSIS IN EXCEL

Part 1

Introduction to Statistics

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14-06-2010

The course:

- **Reminds statistical basics**
- **Gives the methodological tools for the research**
- **Provides practical skill for fast data analysis**

Organization

- 4 sessions (6 hours)
- Lectures are integrated with practical work
- PLEASE: ask questions. Understanding is extremely important for future parts

- Descriptive statistics
- Exploratory analysis
- Discrete probability distribution
- Continues probability distribution

Look for the data:
`http://edu.sablab.net/data/xls`

Frequency distribution
A tabular summary of data showing the number (frequency) of items in each of several nonoverlapping classes.

Marks
A
B
C
B
A
B
B
A
B
C

Frequency distribution:

Mark	Frequency
A	3
B	5
C	2
Total	**10**

Relative frequency distribution:

Mark	Frequency
A	0.3
B	0.5
C	0.2
Total	**1**

Percent frequency distribution:

Mark	Frequency
A	30%
B	50%
C	20%
Total	**100%**

In MS Excel use the following functions:

- **`=COUNTIF(data,element)`** to get number of “elements” found in the “data” area
- **`=SUM(data)`** to get the sum of the values in the “data” area

Example: Pancreatitis Study

The role of smoking in the etiology of pancreatitis has been recognized for many years. To provide estimates of the quantitative significance of these factors, a hospital-based study was carried out in eastern Massachusetts and Rhode Island between 1975 and 1979. **53 patients** who had a hospital discharge diagnosis of **pancreatitis** were included in this unmatched case-control study. The **control group** consisted of 217 patients admitted for **diseases other** than those of the pancreas and biliary tract. Risk factor information was obtained from a standardized interview with each subject, conducted by a trained interviewer.

adapted from Chap T. Le, Introductory Biostatistics

`pancreatitis.xls`

Pancreatitis patients:

Smokers	Ex-smokers	Ex-smokers	Smokers	Smokers	Smokers
Ex-smokers	Smokers	Smokers	Smokers	Smokers	Smokers
Ex-smokers	Smokers	Smokers	Ex-smokers	Smokers	Smokers
Ex-smokers	Ex-smokers	Smokers	Ex-smokers	Smokers	
Smokers	Never	Smokers	Ex-smokers	Ex-smokers	
Smokers	Ex-smokers	Smokers	Smokers	Ex-smokers	
Smokers	Smokers	Smokers	Smokers	Smokers	
Ex-smokers	Smokers	Smokers	Smokers	Smokers	
Smokers	Smokers	Smokers	Smokers	Smokers	
Smokers	Never	Smokers	Smokers	Smokers	

Frequency Distribution

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`pancreatitis.xls`

Frequency distribution:

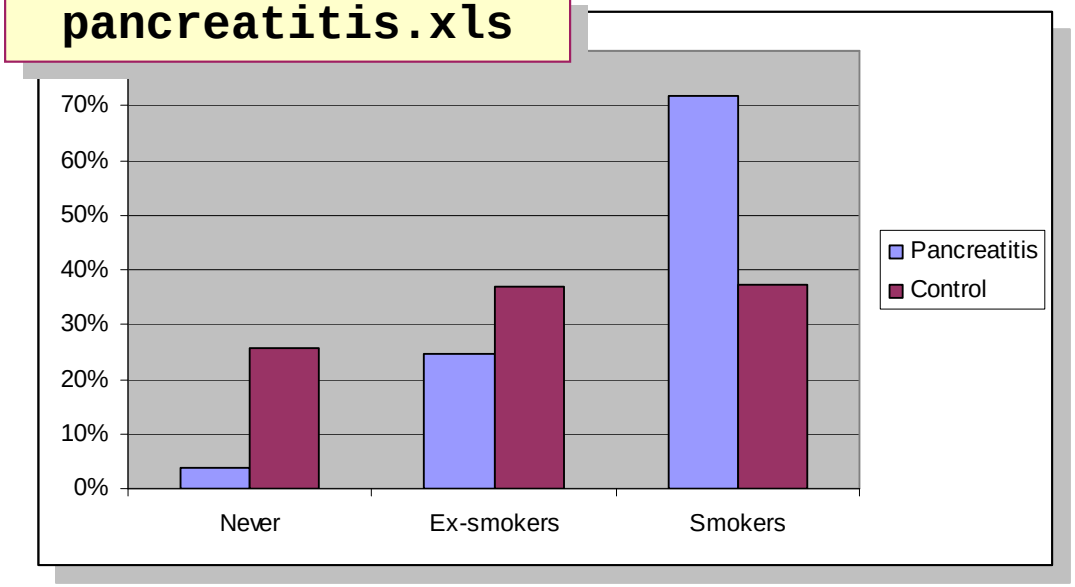
Smoking	Cases	Controls
Never	2	56
Ex-smokers	13	80
Smokers	38	81
Total	53	217

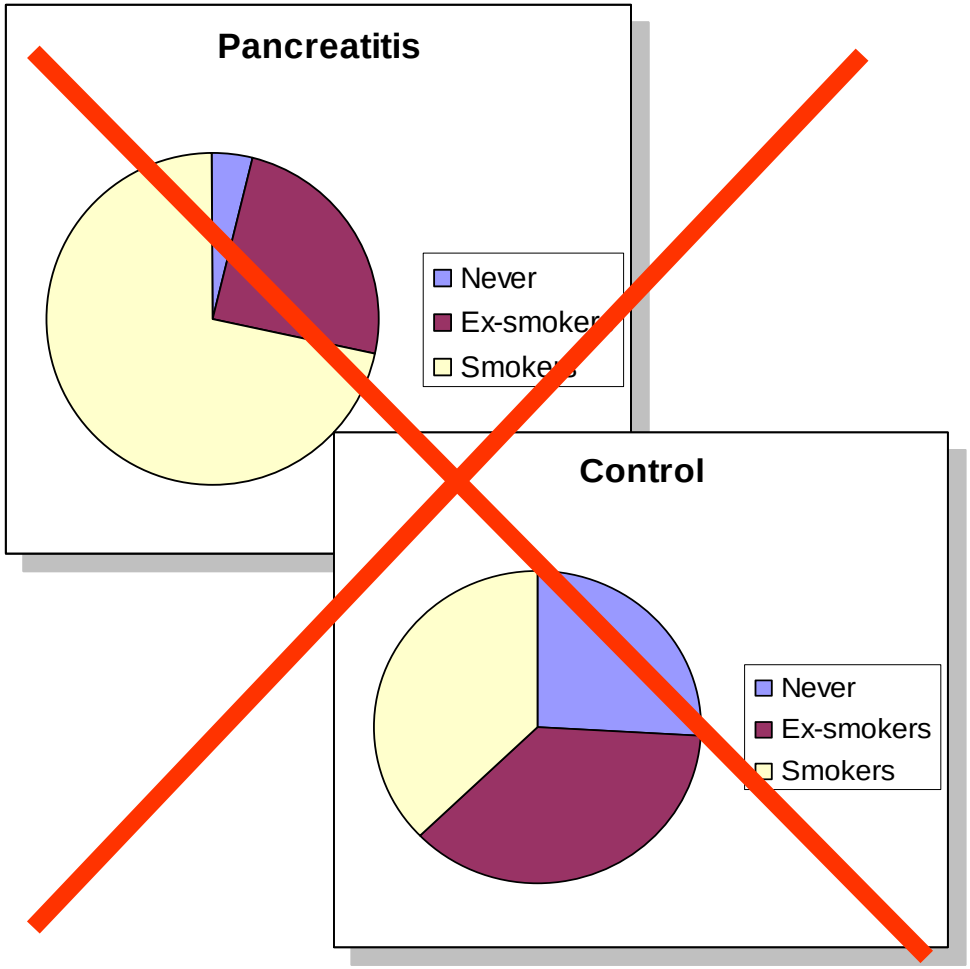
Relative frequency distribution:

Smoking	Cases	Controls
Never	0.038	0.258
Ex-smokers	0.245	0.369
Smokers	0.717	0.373
Total	1	1

Bar and Pie Charts

pancreatitis.xls



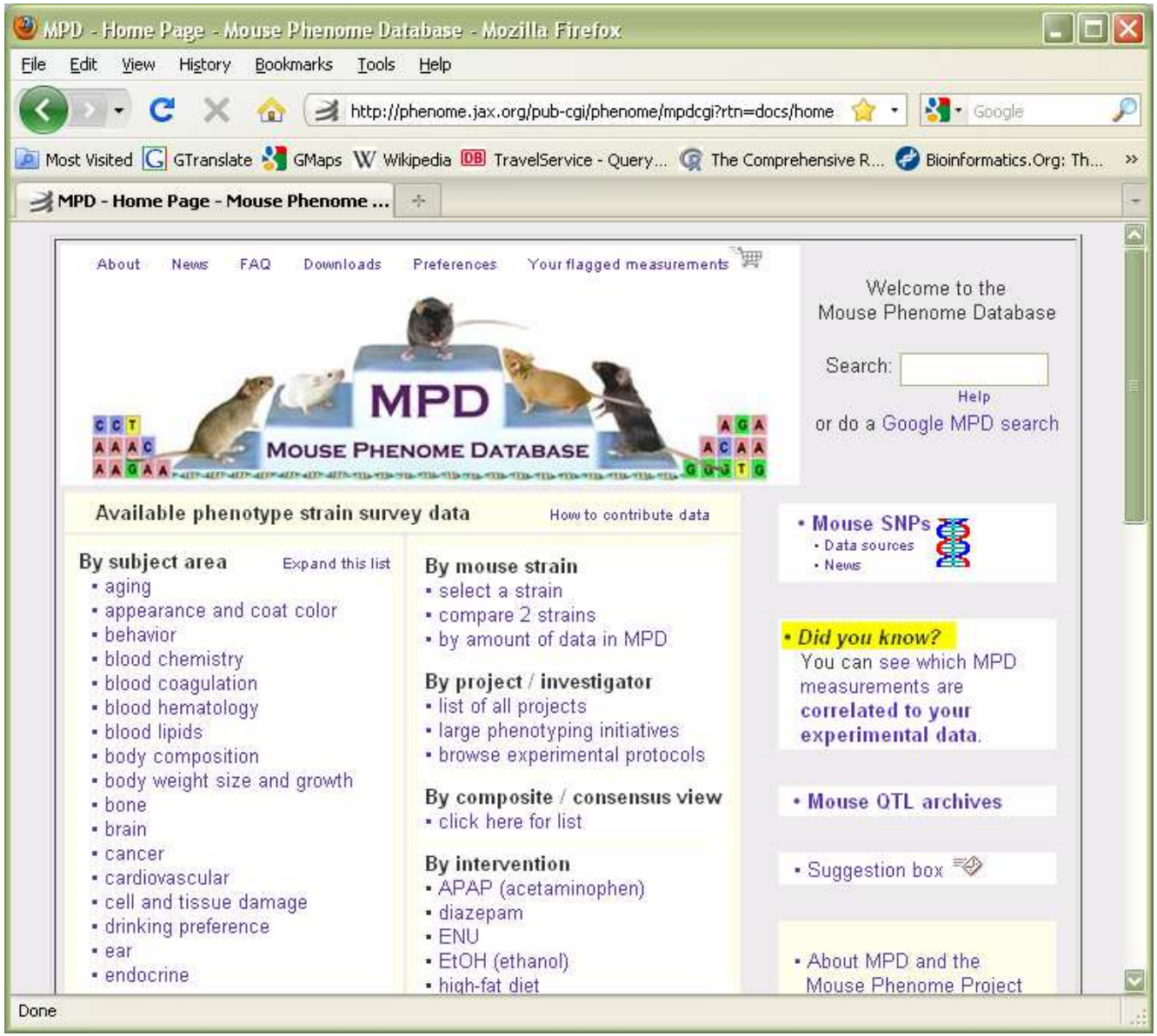


In MS Excel use the following steps:

- Chart Wizard → Columns → Set data range (both columns of Percent freq. distribution)
- Chart Wizard → Pie → Set data range (one columns of Percent freq. distribution)

TABULAR AND GRAPHICAL PRESENTATION

Mice Data Series



Tordoff MG, Bachmanov AA

Survey of calcium & sodium intake and metabolism with bone and body composition data

Project symbol: **Tordoff3**

Accession number: **MPD:103**

`mice.xls`

790 mice from different strains

http://phenome.jax.org

parameter
Starting age
Ending age
Starting weight
Ending weight
Weight change
Bleeding time
Ionized Ca in blood
Blood pH
Bone mineral density
Lean tissues weight
Fat weight

Histogram

The following are weights in grams for 970 mice:

mice.xls

20.5	23.2	24.6	23.5	26	25.9	23.9	22.8	19.9	…
20.8	22.4	26	23.8	26.5	26	22.8	22.9	20.9	…
19.8	22.7	31	22.7	26.3	27.1	18.4	21	18.8	…
21	21.4	25.7	19.7	27	26.2	21.8	22.2	19.2	…
21.9	22.6	23.7	26.2	26	27.5	25	20.9	20.6	…
22.1	20	21.1	24.1	28.8	30.2	20.1	24.2	25.8	…
21.3	21.8	23.7	23.5	28	27.6	21.6	21	21.3	…
20.1	20.8	24.5	23.8	29.5	21.4	21.5	24	21.1	…
18.9	19.5	32.3	28	27.1	28.2	22.9	19.9	20.4	…
21.3	20.6	22.8	25.8	24.1	23.5	24.2	22	20.3	…

Sorted weights show that the values are in the 10 – 49.6 grams.
Let us divide the weight into the “bins”

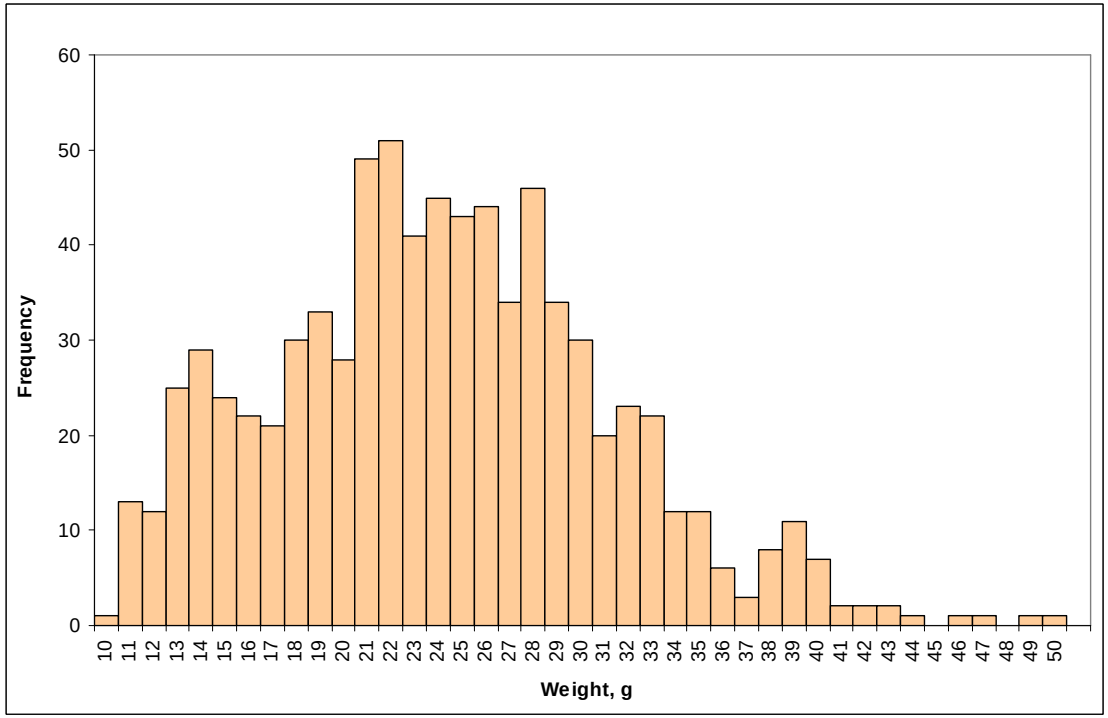
bins

Weight,g	*Frequency*
>=10	1
10-20	237
20-30	417
30-40	124
40-50	11
More	0

Histogram

Now, let us use bin-size = 1 gram

Bin	*Frequency*
10	1
11	13
12	12
13	25
14	29
...	...
46	1
47	1
48	0
49	1
50	1
More	0

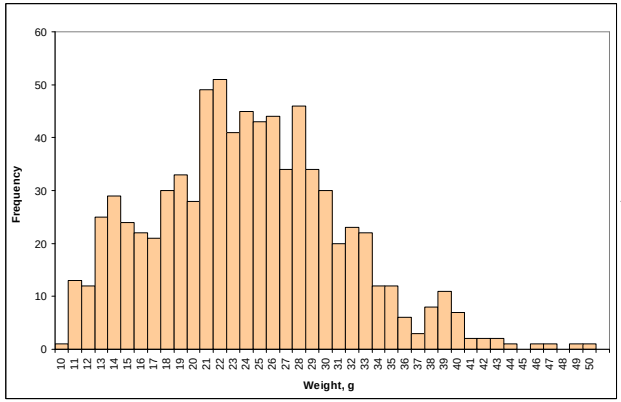


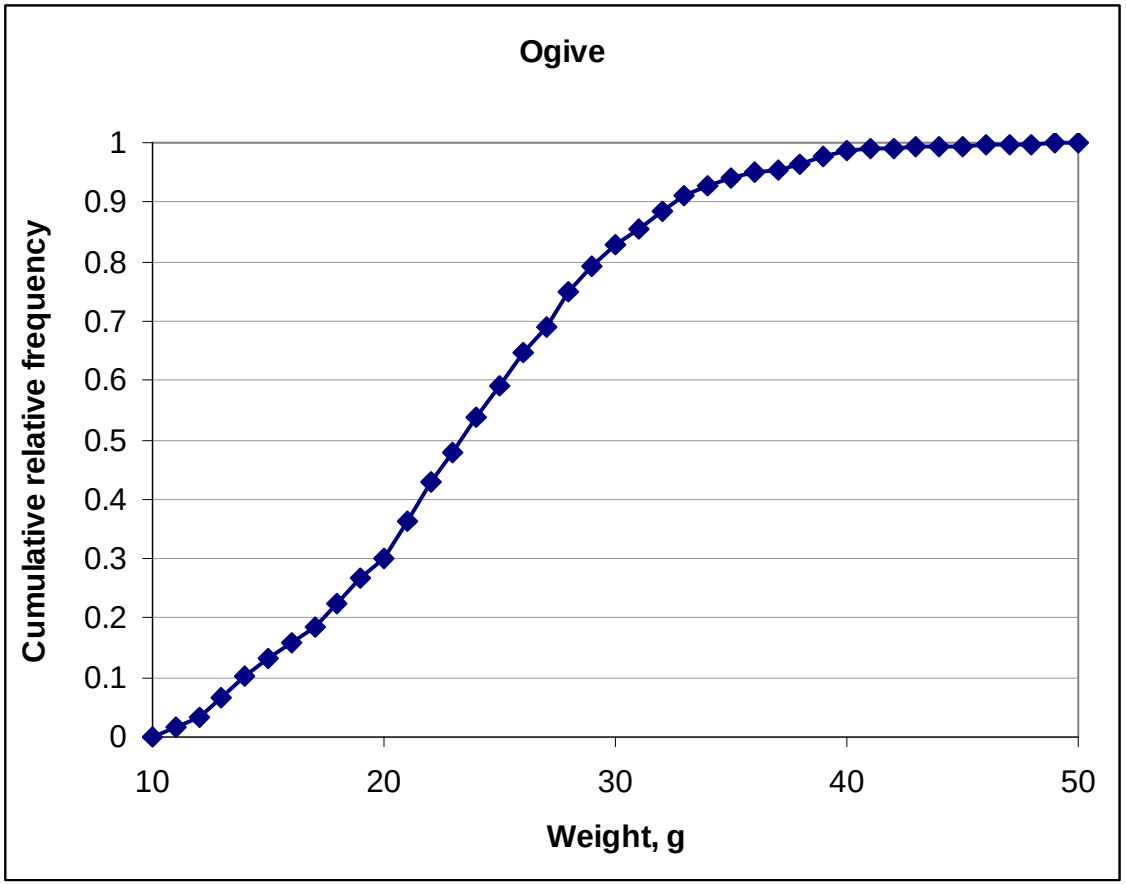
In Excel use the following steps:

- Specify the column of bins (interval) upper-limits
- Tools → Data Analysis → Histrogram → select the input data, bins, and output (Analysis ToolPak should be installed)
- use Chart Wizard → Columns to visualize the results

Cumulative Frequency Distribution

Cumulative frequency distribution
A tabular summary of quantitative data showing the number of items with values less than or equal to the upper class limit of each class.

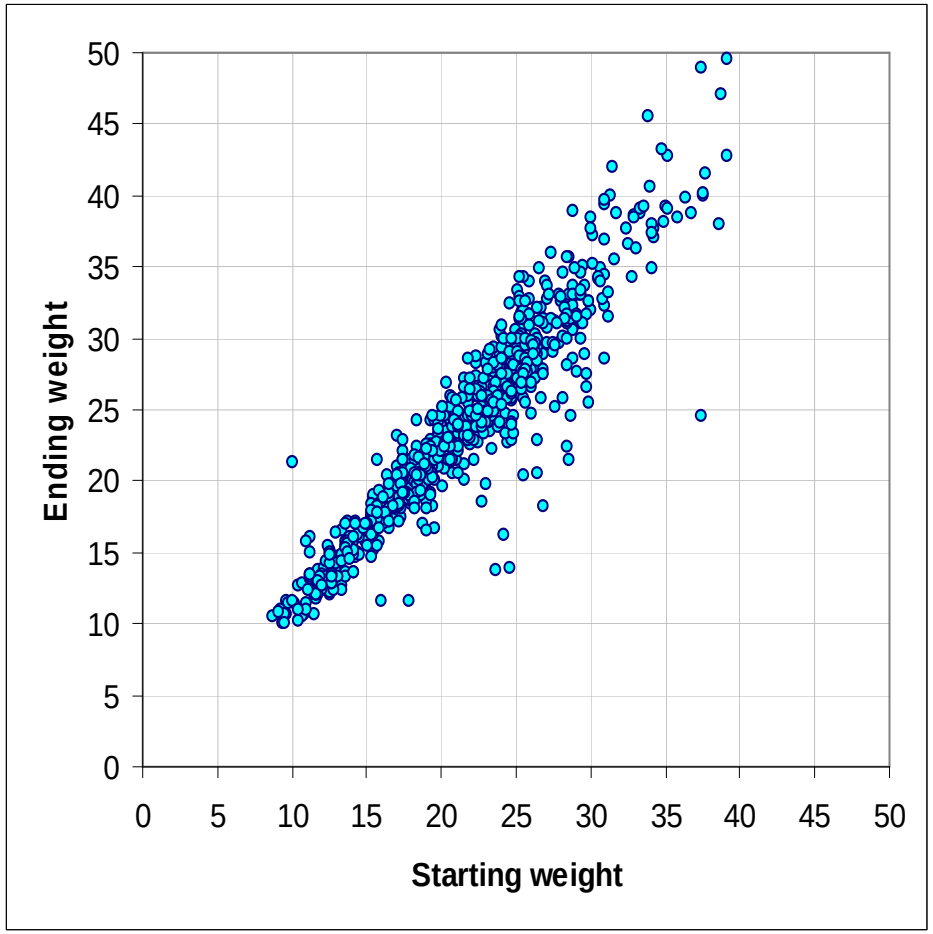




Scatter Plot

mice.xls

Let us look on mutual dependency of the Starting and Ending weights.



In Excel use the following steps:

- Select the data region
- Use Chart Wizard → XY (Scatter)

Crosstabulation

pancreatitis.xls

Smoking	*Disease* other	pancreatitis	Total
Ex-smokers	80	13	93
Never	56	2	58
Smokers	81	38	119
Total	217	53	**270**

In Excel use the following steps:

- Data → Pivot Table and PivotChart → MS Office list + Pivot Table
- Set the range, including the headers of the data
- Select output and set layout by drag-and-dropping the names into the table

Population parameter
A numerical value used as a summary measure for a population (e.g., the population mean μ, variance σ^2, standard deviation σ)

POPULATION

μ – mean
σ^2 – variance
N – number of elements (usually $N=\infty$)

SAMPLE

$m, \bar{x}$ – mean
s^2 – variance
n – number of elements

Sample statistic
A numerical value used as a summary measure for a sample (e.g., the sample mean m, the sample variance s^2, and the sample standard deviation s)

All existing laboratory
Mus musculus

mice.xls

790 mice from different strains

http://phenome.jax.org

ID	Strain	Sex	Starting age	Ending age	Starting weight	Ending weight	Weight change	Bleeding time	Ionized Ca in blood	Blood pH	Bone mineral density	Lean tissues weight	Fat weight
1	129S1/SvImJ	f	66	116	19.3	20.5	1.062	64	1.2	7.24	0.0605	14.5	4.4
2	129S1/SvImJ	f	66	116	19.1	20.8	1.089	78	1.15	7.27	0.0553	13.9	4.4
3	129S1/SvImJ	f	66	108	17.9	19.8	1.106	90	1.16	7.26	0.0546	13.8	2.9
368	129S1/SvImJ	f	72	114	18.3	21	1.148	65	1.26	7.22	0.0599	15.4	4.2
369	129S1/SvImJ	f	72	115	20.2	21.9	1.084	55	1.23	7.3	0.0623	15.6	4.3
370	129S1/SvImJ	f	72	116	18.8	22.1	1.176		1.21	7.28	0.0626	16.4	4.3
371	129S1/SvImJ	f	72	119	19.4	21.3	1.098	49	1.24	7.24	0.0632	16.6	5.4
372	129S1/SvImJ	f	72	122	18.3	20.1	1.098	73	1.17	7.19	0.0592	16	4.1
4	129S1/SvImJ	f	66	109	17.2	18.9	1.099	41	1.25	7.29	0.0513	14	3.2
5	129S1/SvImJ	f	66	112	19.7	21.3	1.081	129	1.14	7.22	0.0501	16.3	5.2
10	129S1/SvImJ	m	66	112	24.3	24.7	1.016	119	1.13	7.24	0.0533	17.6	6.8
364	129S1/SvImJ	m	72	114	25.3	27.2	1.075	64	1.25	7.27	0.0596	19.3	5.8
365	129S1/SvImJ	m	72	115	21.4	23.9	1.117	48	1.25	7.28	0.0563	17.4	5.7
366	129S1/SvImJ	m	72	118	24.5	26.3	1.073	59	1.25	7.26	0.0609	17.8	7.1
367	129S1/SvImJ	m	72	122	24	26	1.083	69	1.29	7.26	0.0584	19.2	4.6
6	129S1/SvImJ	m	66	116	21.6	23.3	1.079	78	1.15	7.27	0.0497	17.2	5.7
7	129S1/SvImJ	m	66	107	22.7	26.5	1.167	90	1.18	7.28	0.0493	18.7	7
8	129S1/SvImJ	m	66	108	25.4	27.4	1.079	35	1.24	7.26	0.0538	18.9	7.1
9	129S1/SvImJ	m	66	109	24.4	27.5	1.127	43	1.29	7.29	0.0539	19.5	7.1

NUMERICAL MEASURES

Measures of Location

Mean
A measure of central location computed by summing the data values and dividing by the number of observations.

Median
A measure of central location provided by the value in the middle when the data are arranged in ascending order.

Mode
A measure of location, defined as the value that occurs with greatest frequency.

$$\bar{x} = m = \frac{\sum x_i}{n}$$

$$\mu = \frac{\sum x_i}{N}$$

$$p = \frac{\sum (x_i = true)}{n}$$

Weight
12
16
19
22
23
23
24
32
36
42
63
68

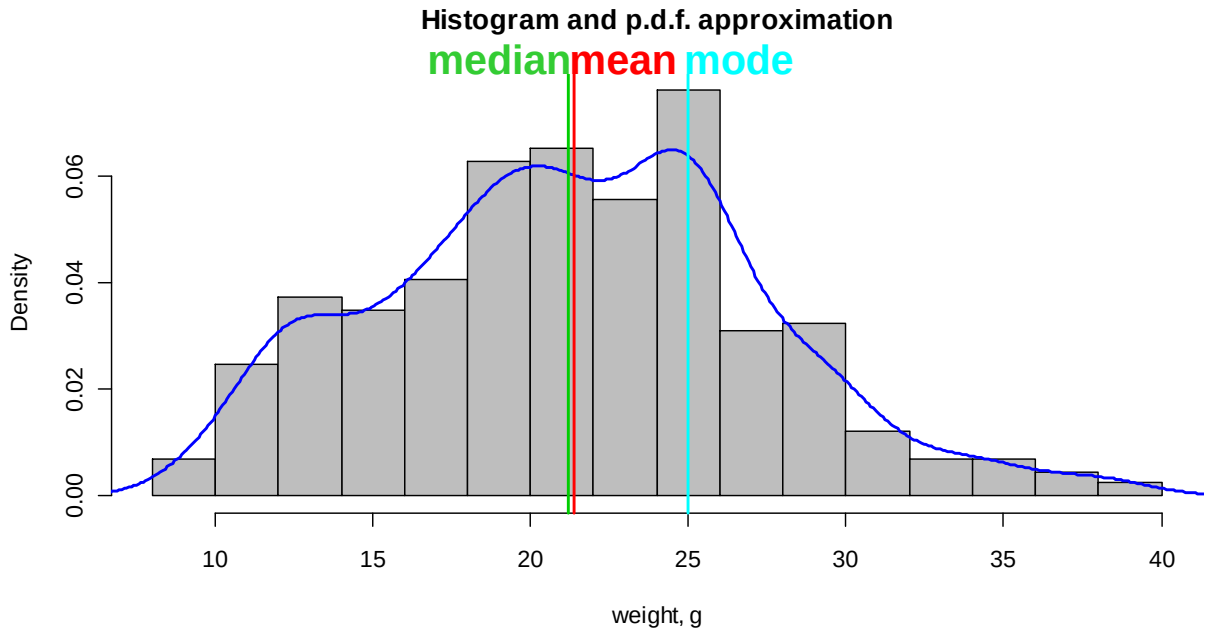
Mode = 23

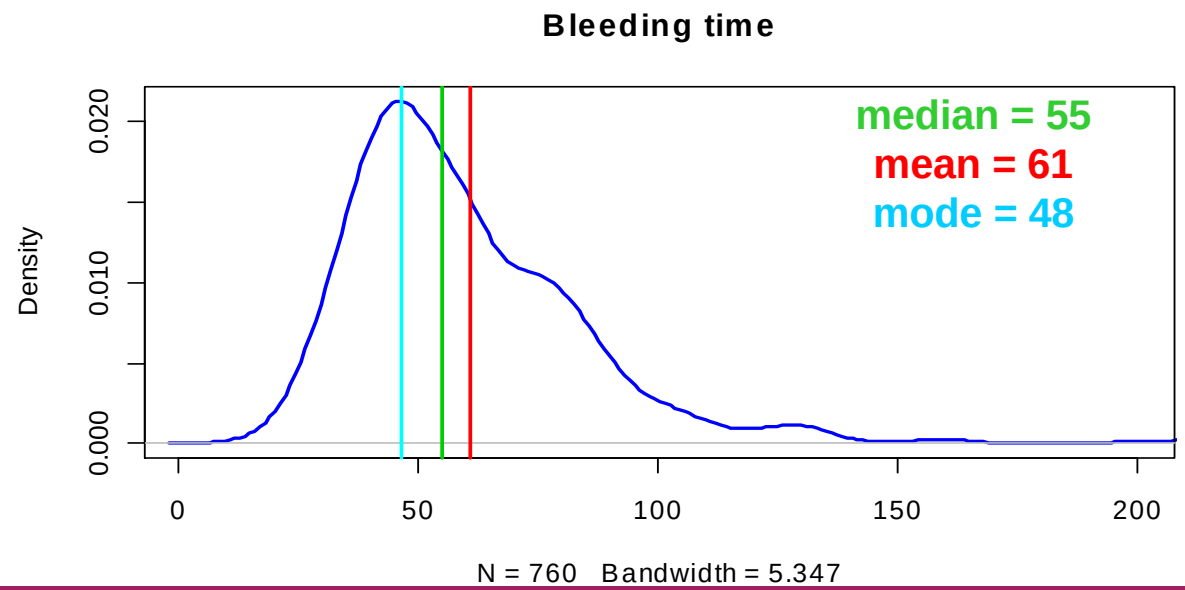
Median = 23.5

Mean = 31.7

mice.xls

Female proportion
$p_f = 0.501$





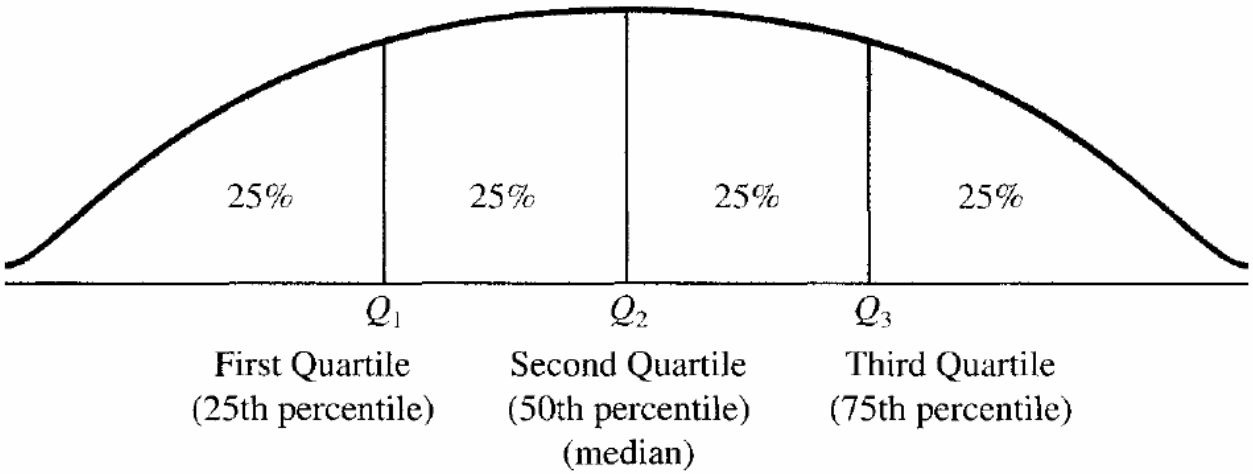
In Excel use the following functions:

- **= AVERAGE(data)**
- **= MEDIAN(data)**
- **= MODE(data)**

Quantiles, Quartiles and Percentiles

Percentile
A value such that at least p% of the observations are less than or equal to this value, and at least (100-p)% of the observations are greater than or equal to this value. The 50-th percentile is the ***median***.

Quartiles
The 25th, 50th, and 75th percentiles, referred to as the first quartile, the second quartile (median), and third quartile, respectively.



In Excel use the following functions:

- `=PERCENTILE(data,p)`

Weight	12	16	19	22	23	23	24	32	36	42	63	68

$Q_1 = 21$ $Q_2 = 23.5$ $Q_3 = 39$

Interquartile range (IQR)
A measure of variability, defined to be the difference between the third and first quartiles.

$$IQR = Q_3 - Q_1$$

Variance
A measure of variability based on the squared deviations of the data values about the mean.

population

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{N}$$

sample

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

Standard deviation
A measure of variability computed by taking the positive square root of the variance.

$$Sample\ standard\ deviation = s = \sqrt{s^2}$$

$$Population\ standard\ deviation = \sigma = \sqrt{\sigma^2}$$

Weight	12	16	19	22	23	23	24	32	36	42	63	68

IQR = 18 *Variance* = 320.2 *St. dev.* = 17.9

In Excel use the following functions:

- **=VAR(data), =STDEV(data)**

Coefficient of variation
A measure of relative variability computed by dividing the standard deviation by the mean.

Weight	12	16	19	22	23	23	24	32	36	42	63	68

$$\left(\frac{Standard\ deviation}{Mean}\times 100\right)\%$$

$CV = 57\%$

Median absolute deviation (MAD)
MAD is a robust measure of the variability of a univariate sample of quantitative data.

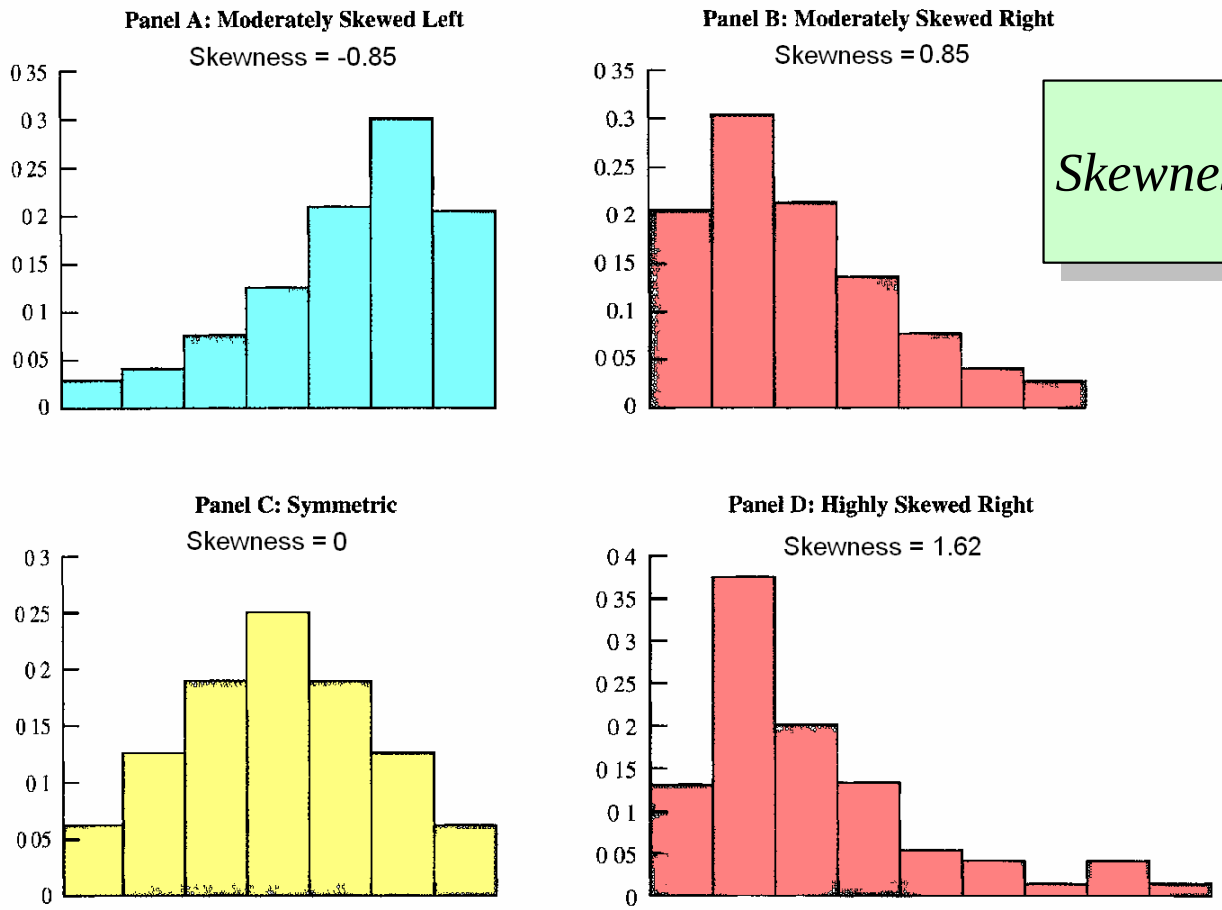
$$MAD = median\left(\left|x_i - median(x)\right|\right)$$

Set 1	Set 2
23	23
12	12
22	22
12	12
21	21
18	81
22	22
20	20
12	12
19	19
14	14
13	13
17	17

	Set 1	Set 2
Mean	17.3	22.2
Median	18	19
St.dev.	4.23	18.18
MAD	5.93	5.93

Skewness

A measure of the shape of a data distribution. Data skewed to the left result in negative skewness; a symmetric data distribution results in zero skewness; and data skewed to the right result in positive skewness.



$$Skewness = \frac{n}{(n-1)(n-2)} \sum_i \left(\frac{x_i - \bar{x}}{s} \right)^3$$

adapted from Anderson et al Statistics for Business and Economics

z-score

z-score
A value computed by dividing the deviation about the mean (x_i - x) by the standard deviation s. A ***z-score*** is referred to as a standardized value and denotes the number of standard deviations x_i is from the mean.

$$z_i = \frac{x_i - \bar{x}}{s}$$

Weight	z-score
12	-1.10
16	-0.88
19	-0.71
22	-0.54
23	-0.48
23	-0.48
24	-0.43
32	0.02
36	0.24
42	0.58
63	1.75
68	2.03

Chebyshev's theorem
For **any data set**, at least **$(1 - 1/z^2)$** of the data values must be within z standard deviations from the mean, where z – any value > 1.

For ANY distribution:

- At least **75 %** of the values are within **z = 2** standard deviations from the mean
- At least **89 %** of the values are within **z = 3** standard deviations from the mean
- At least **94 %** of the values are within **z = 4** standard deviations from the mean
- At least **96%** of the values are within **z = 5** standard deviations from the mean

Detection of Outliers

For bell-shaped distributions:

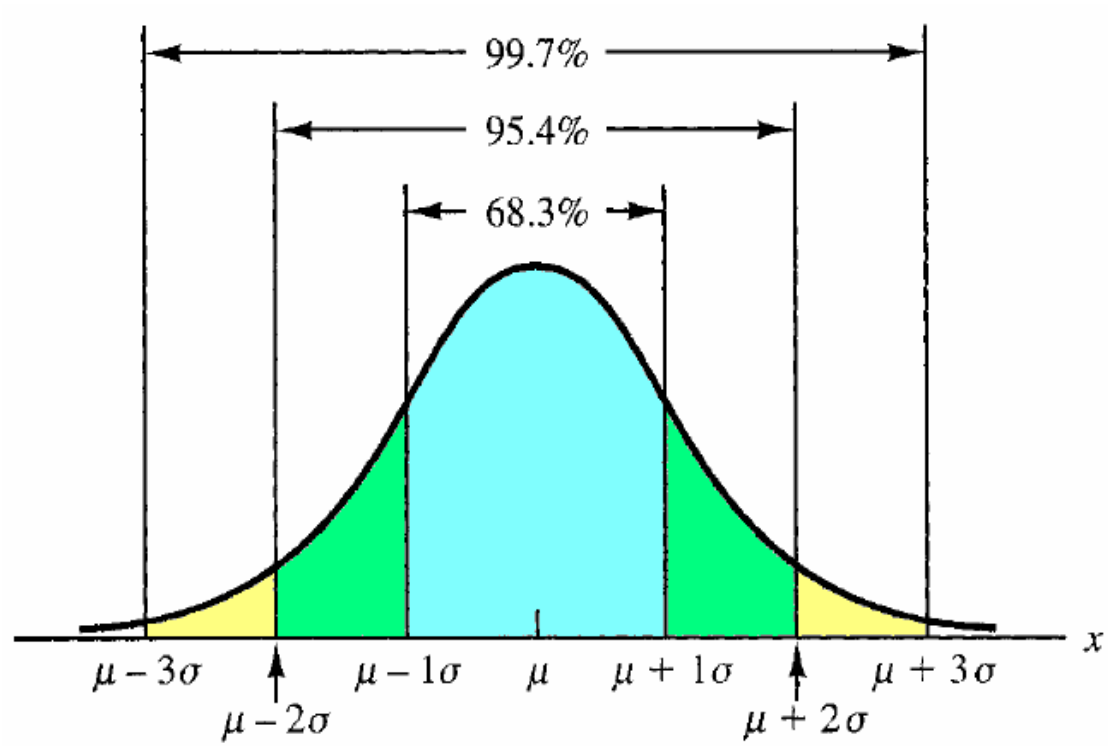
- Approximately 68 % of the values are within 1 st.dev. from mean
- Approximately 95 % of the values are within 2 st.dev. from mean
- Almost all data points are inside 3 st.dev. from mean

Outlier
An unusually small or unusually large data value.

For bell-shaped distributions data points with |z|>3 can be considered as outliers.

Weight	z-score
23	0.04
12	-0.53
22	-0.01
12	-0.53
21	-0.06
81	**3.10**
22	-0.01
20	-0.11
12	-0.53
19	-0.17
14	-0.43
13	-0.48
17	-0.27

Example: Gaussian distribution



Five-number summary
An exploratory data analysis technique that uses five numbers to summarize the data: smallest value, first quartile, median, third quartile, and largest value

children.xls

Min. :	12
Q_1 :	25
Median:	32
Q_3 :	46
Max. :	79

In Excel use:

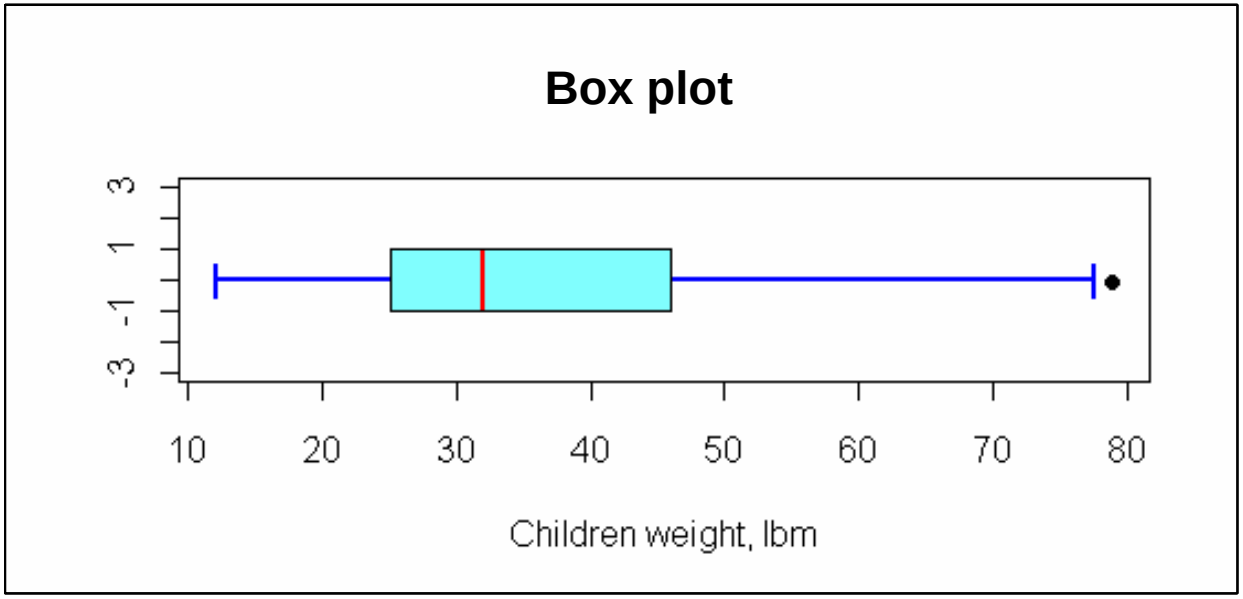
- Tool → Data Analysis → Descriptive Statistics

Box plot
A graphical summary of data based on a five-number summary

In Excel use (indirect):

- Chart Wizard → Stock → Open-high-low-close

open	Q3
high	Q3+1.5*IQR
low	Q1-1.5*IQR
close	Q1



Example: Mice Weight

Example
Build a box plot for weights of male and female mice

mice.xls

1. Build 5 number summaries for males and females

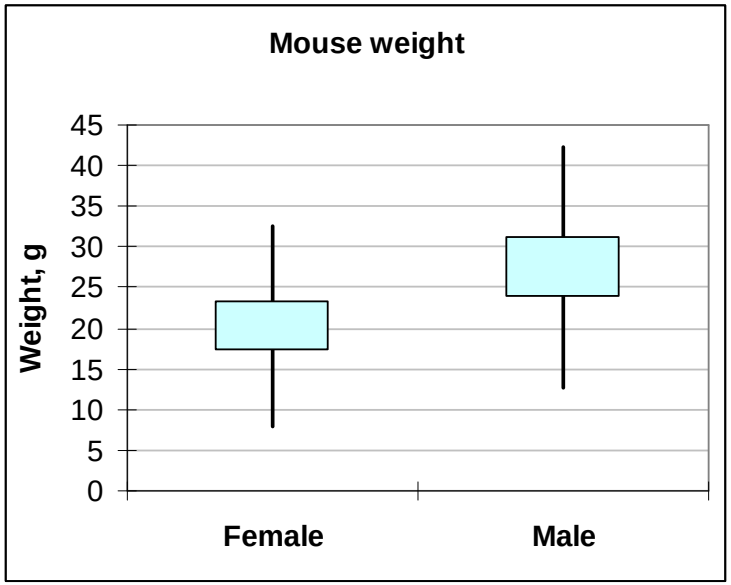
	Female	Male
Min	10.0	12.0
Q1	17.2	23.8
Q2	20.7	27.1
Q3	23.3	31.2
Max	41.5	49.6

2. Combine the numbers into the following order

open	Q3
high	Q3+min(1.5*(Q3-Q1),Max)
low	Q1-max(1.5*(Q3-Q1),Min)
close	Q1

In Excel use:

- Chart Wizard → Stock → Open-high-low-close
- Put “series-in-rows”
- Adjust colors, etc



Measure of Association between 2 Variables

Covariance
A measure of linear association between two variables. Positive values indicate a positive relationship; negative values indicate a negative relationship.

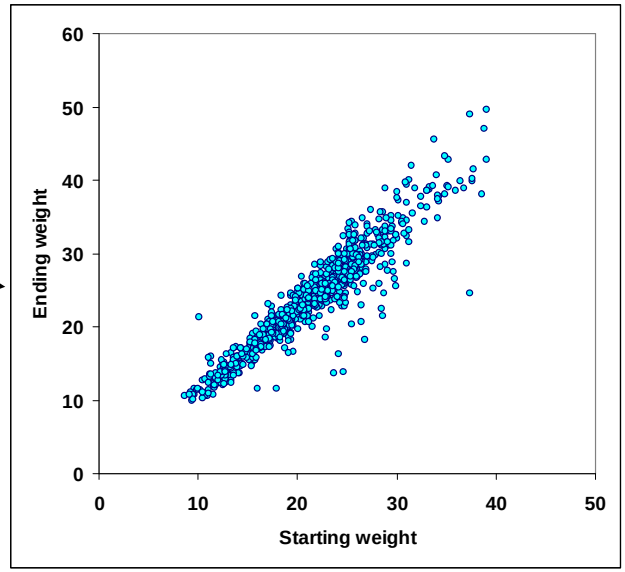
population

$$\sigma_{xy} = \frac{\sum (x_i - \mu_x)(y_i - \mu_y)}{N}$$

sample

$$s_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n-1}$$

mice.xls



Ending weight vs. Starting weight

In Excel use function:

- `=COVAR(data)`

s_{xy} = 39.8

hard to interpret

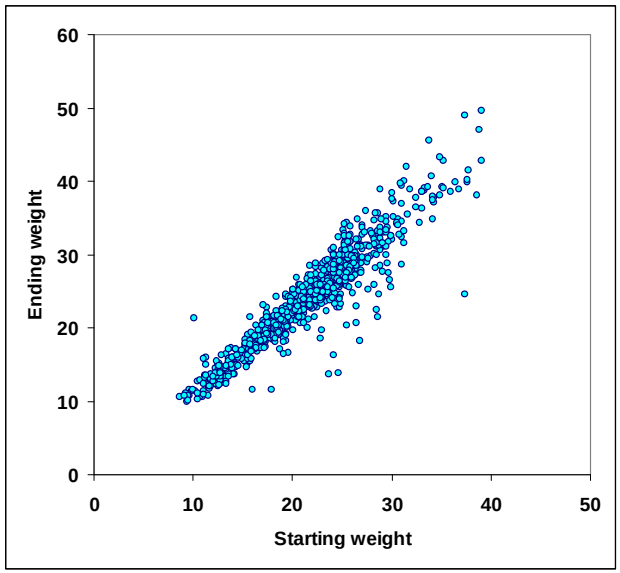
Correlation (Pearson product moment correlation coefficient)
A measure of linear association between two variables that takes on values between -1 and +1. Values near +1 indicate a strong positive linear relationship, values near -1 indicate a strong negative linear relationship; and values near zero indicate the lack of a linear relationship.

population

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sigma_x \sigma_y N}$$

sample

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{s_x s_y (n-1)}$$



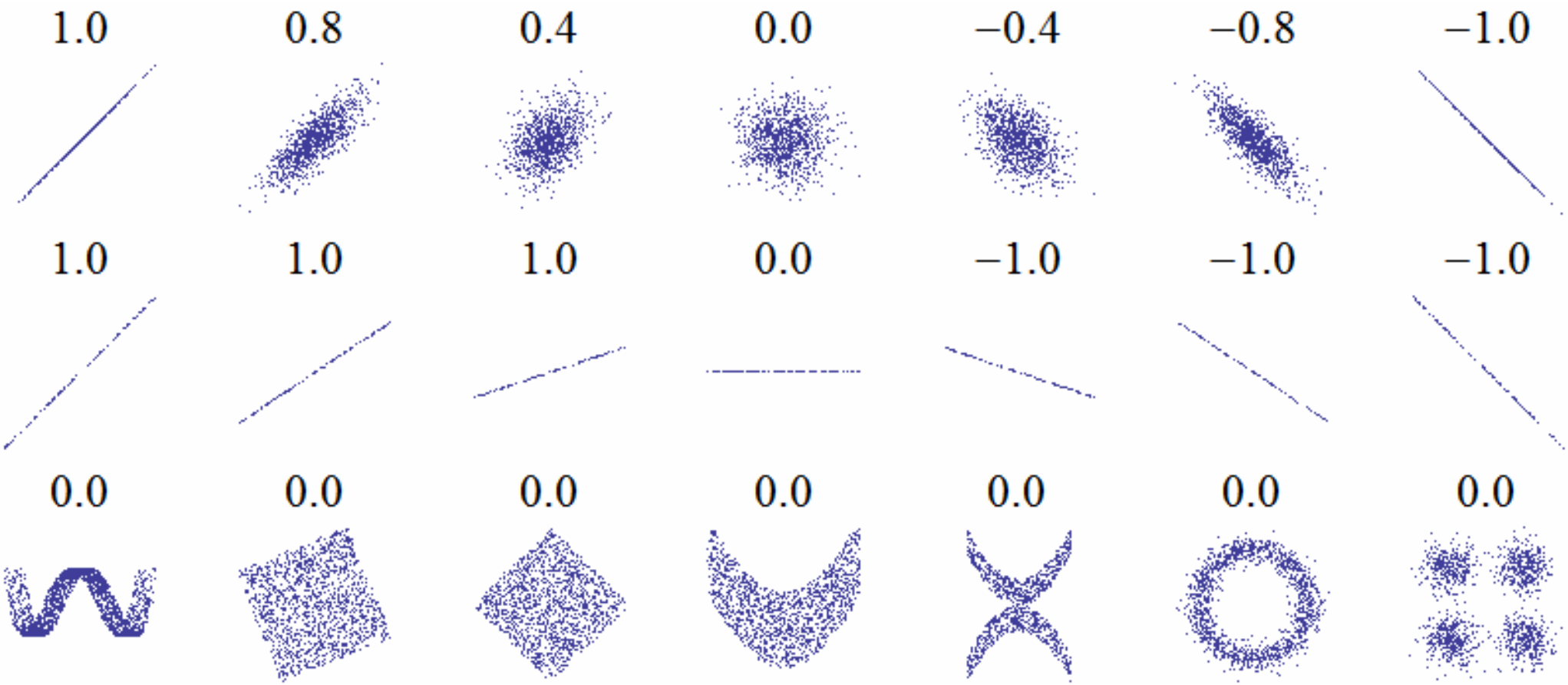
In Excel use function:

- `=CORREL(data)`

r_{xy} = 0.94

mice.xls

Correlation Coefficient



Wikipedia

? If we have only 2 data points in *x* and y datasets, what values would you expect for correlation b/w *x* and y ?

- **Discrete and continuous probability distributions**
  - discrete probability distribution
  - continuous probability distribution
  - normal probability distribution

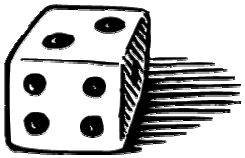
Random variable
A numerical description of the outcome of an experiment.

A random variable is always a numerical measure.

Discrete random variable
A random variable that may assume either a finite number of values or an infinite sequence of values.

Continuous random variable
A random variable that may assume any numerical value in an interval or collection of intervals.

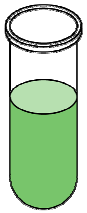
Roll a die



Number of calls to a reception per hour

Time between calls to a reception

Volume of a sample in a tube



Weight, height, blood pressure, etc

Discrete Probability Distribution

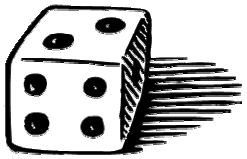
Probability distribution
A description of how the probabilities are distributed over the values of the random variable.

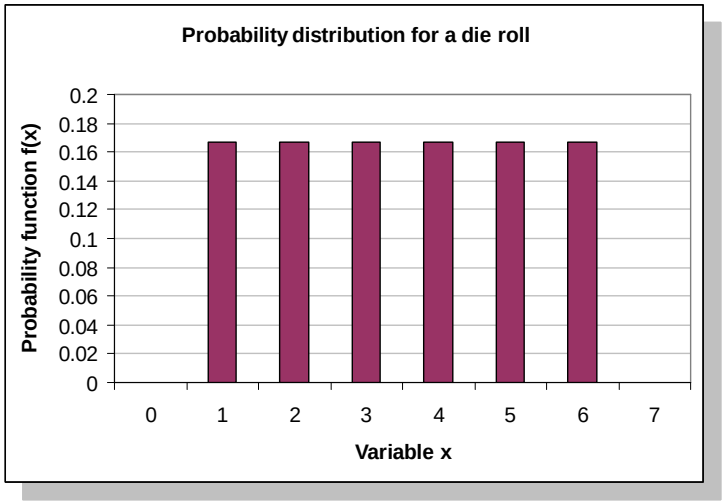
Probability function
A function, denoted by *f(x)*, that provides the probability that *x* assumes a particular value for a discrete random variable.

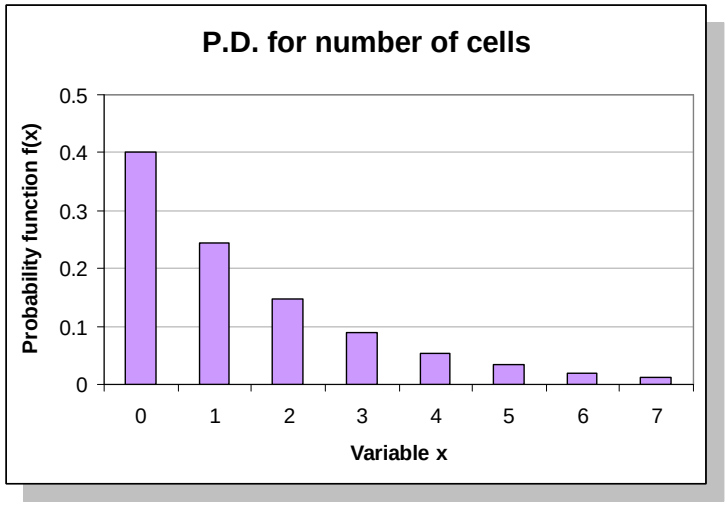
Number of cells under microscope
Random variable X:
x = 0
x = 1
x = 2
x = 3
…

$$f(x) \geq 0$$
$$\sum f(x) = 1$$

Roll a die
Random variable X:
x = 1
x = 2
x = 3
x = 4
x = 5
x = 6

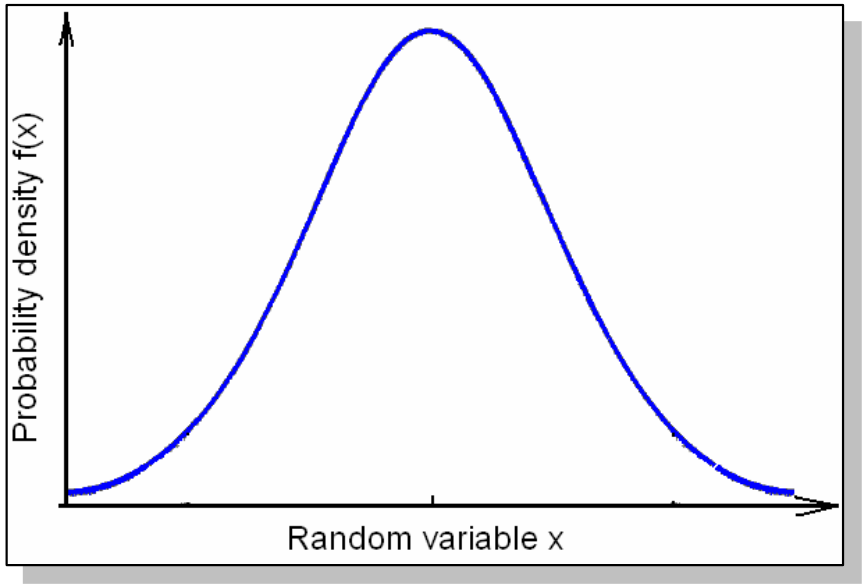


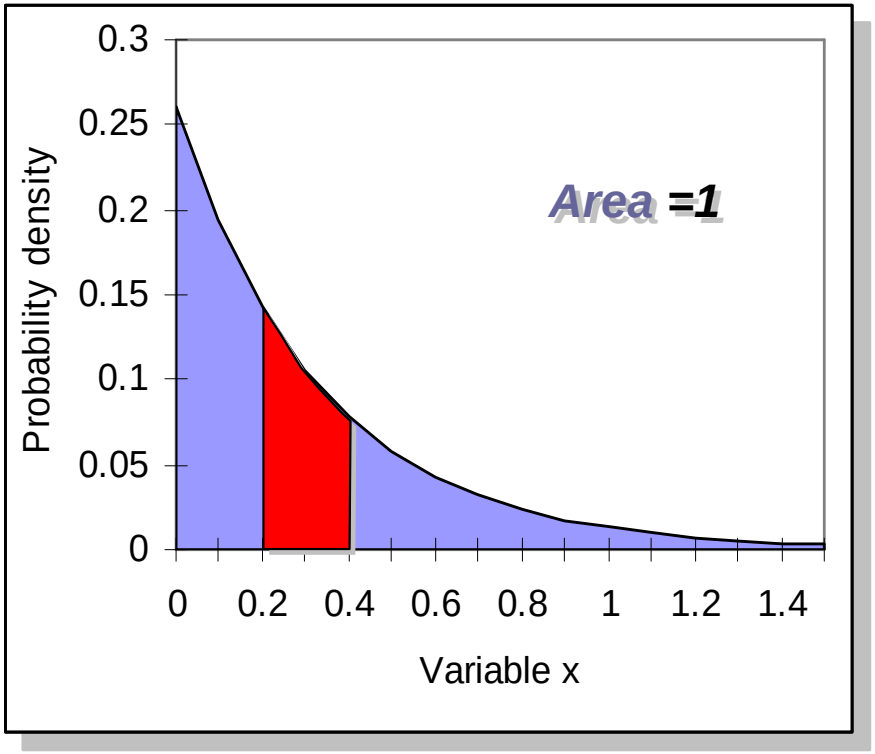




Probability density function

A function used to compute probabilities for a continuous random variable. The area under the graph of a probability density function over an interval represents probability.



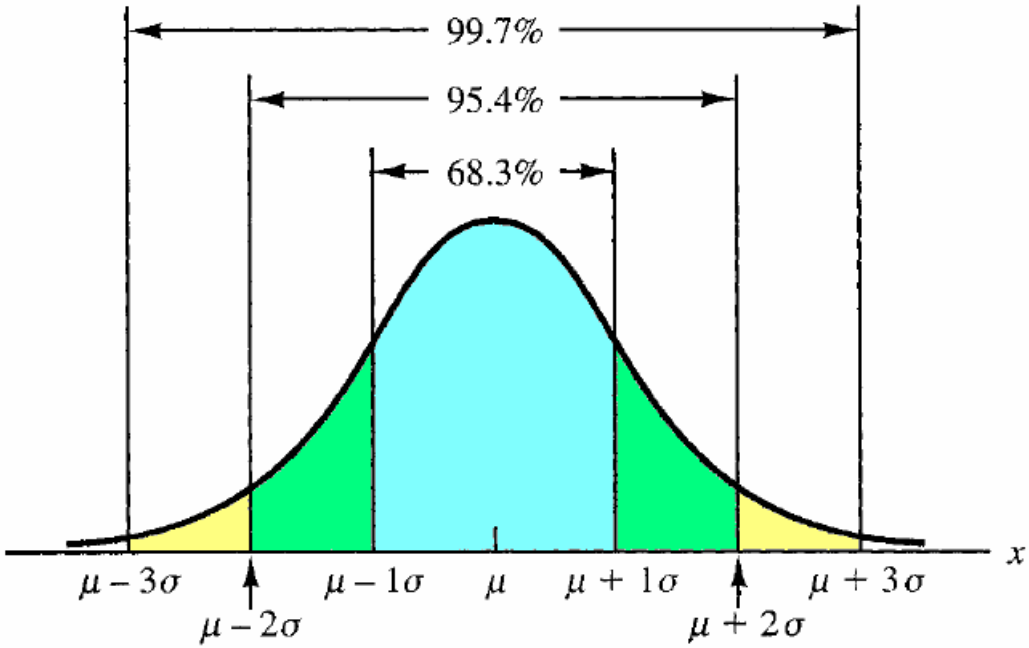


$$\int_x f(x) = 1$$

Normal Probability Distribution

Normal probability distribution
A continuous probability distribution. Its probability density function is bell shaped and determined by its mean μ and standard deviation σ.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



In Excel use the function:

- **`= NORMDIST(x,m,s,false)`** for probability density function
- **`= NORMDIST(x,m,s,true)`** for cumulative probability function of normal distribution (area from left to x)

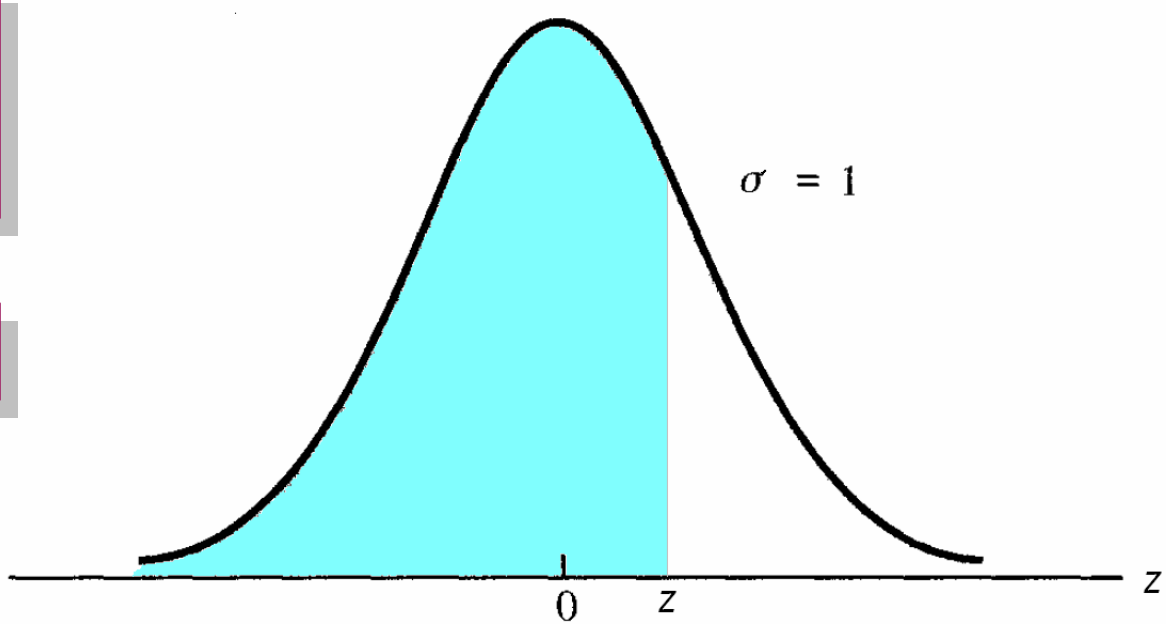
Standard Normal Probability Distribution

Standard normal probability distribution
A normal distribution with a mean of zero and a standard deviation of one.

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$z = \frac{x - \mu}{\sigma}$$

$$x = z\sigma + \mu$$



In Excel use the function:

- `= NORMSDIST(z)`

Example

Assume that you have developed an extremely efficient chemical treatment for glioblastoma. During tests on animal models it was found that the substance X, which you use, is able to kill all tumor cells (theoretically), but being given at high concentration it leads to the death of a patient due to intoxication. As the survived cancer cells fast evolve into resistant form, the efficiency of the treatment is significantly reduced if the second course is given. Therefore the treatment should be performed in one injection.

The experimental data suggest that the average concentration needed for the positive treatment is 1 μg/kg. The concentration needed for effective treatment is, of course, a random variable. Being presented in log10 scale and in g/kg, it can be approximated by a normal random variable with mean of –6 and standard deviation of 0.4.

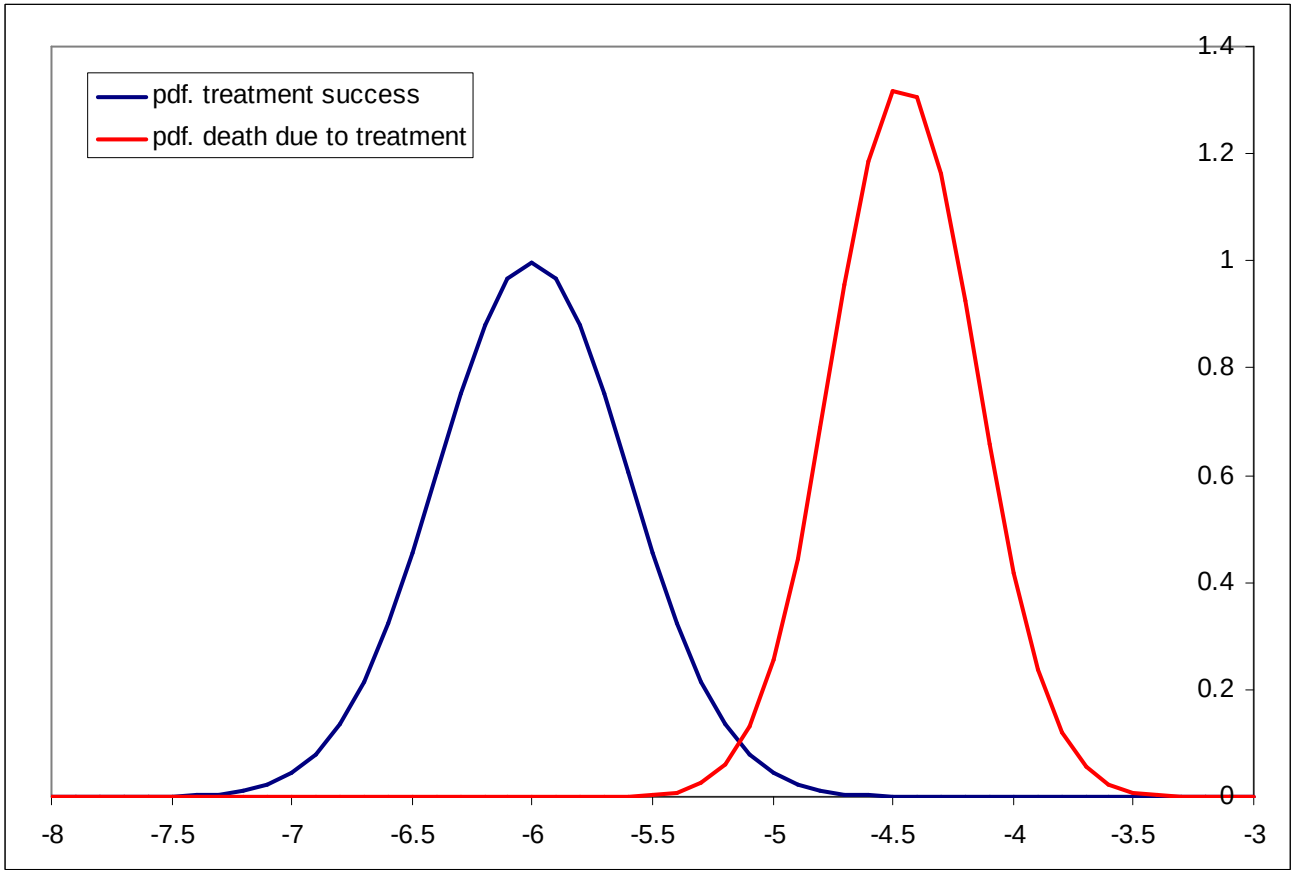
The 50% lethal dose for human is 35 μg/kg. And the tests on animals suggest that in log10 scale it has a normal distribution as well with the standard deviation of 0.3.

parameter	**ug/kg**	**log scale**
mean positive treatment	1	-6
std positive treatment	x	0.4
mean lethal dose	35	-4.456
std lethal dose	x	0.3

CONTINUOUS PROBABILITY DISTRIBUTIONS

Dose Selection

parameter	ug/kg	log scale
mean positive treatment	1	-6
std positive treatment	x	0.4
mean lethal dose	35	-4.456
std lethal dose	x	0.3



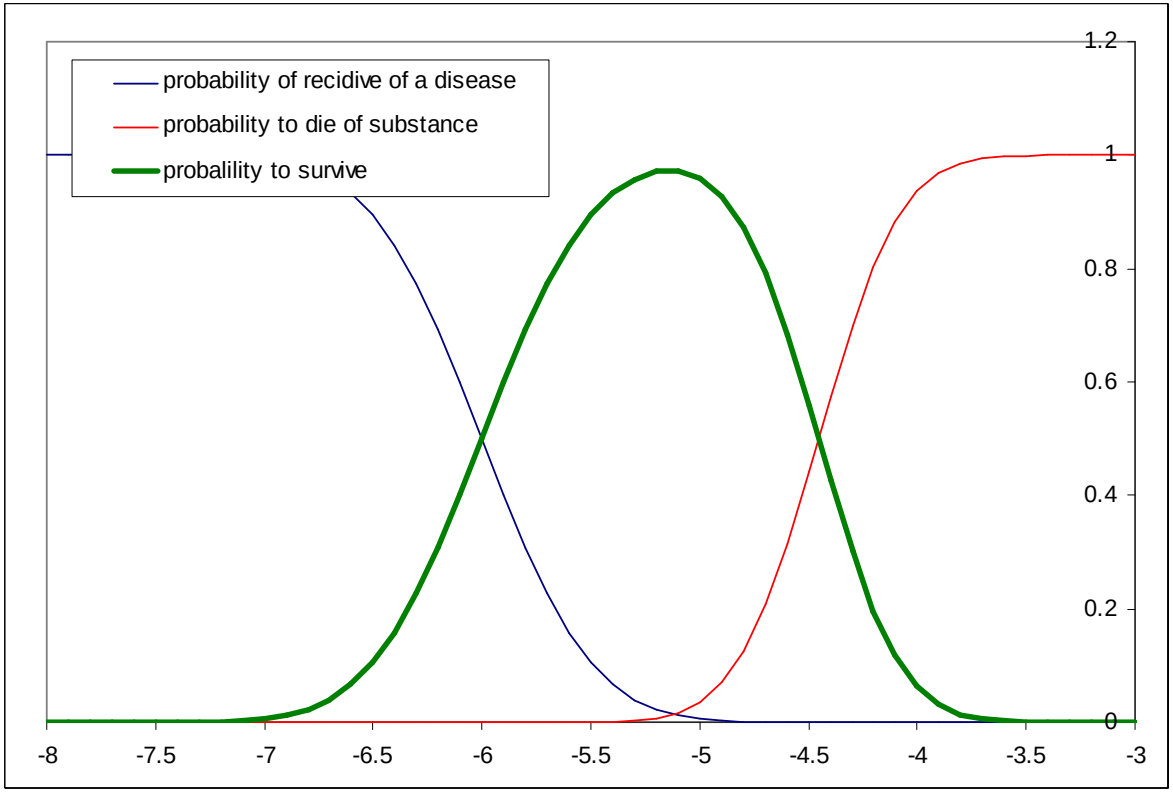
In Excel use the function:

- **= NORMDIST(x,mean,std,FALSE)**

Dose Selection

- Probability to die from disease = inverse probability to treat
- Over-dose and disease behaviors are independent =>

P(survive) = P(heal disease) * P(survive treatment)



In Excel use the function:

- **`= NORMDIST(x,mean,std,TRUE)`**

- **Sampling distribution**
  - sample and population and their parameters
  - central limit theorem
  - types of sampling

SAMPLING DISTRIBUTION

Parameters

Population parameter
A numerical value used as a summary measure for a population (e.g., the mean μ, variance σ^2, standard deviation σ, *proportion* π)

POPULATION

μ – mean
σ^2 – variance
N – number of elements (usually $N=\infty$)

SAMPLE

$m, \bar{x}$ – mean
s^2 – variance
n – number of elements

Sample statistic
A numerical value used as a summary measure for a sample (e.g., the sample mean m, the sample variance s^2, and the sample standard deviation s)

All existing laboratory
Mus musculus

mice.xls

790 mice from different strains

http://phenome.jax.org

ID	Strain	Sex	Starting age	Ending age	Starting weight	Ending weight	Weight change	Bleeding time	Ionized Ca in blood	Blood pH	Bone mineral density	Lean tissues weight	Fat weight
1	129S1/SvImJ	f	66	116	19.3	20.5	1.062	64	1.2	7.24	0.0605	14.5	4.4
2	129S1/SvImJ	f	66	116	19.1	20.8	1.089	78	1.15	7.27	0.0553	13.9	4.4
3	129S1/SvImJ	f	66	108	17.9	19.8	1.106	90	1.16	7.26	0.0546	13.8	2.9
368	129S1/SvImJ	f	72	114	18.3	21	1.148	65	1.26	7.22	0.0599	15.4	4.2
369	129S1/SvImJ	f	72	115	20.2	21.9	1.084	55	1.23	7.3	0.0623	15.6	4.3
370	129S1/SvImJ	f	72	116	18.8	22.1	1.176		1.21	7.28	0.0626	16.4	4.3
371	129S1/SvImJ	f	72	119	19.4	21.3	1.098	49	1.24	7.24	0.0632	16.6	5.4
372	129S1/SvImJ	f	72	122	18.3	20.1	1.098	73	1.17	7.19	0.0592	16	4.1
4	129S1/SvImJ	f	66	109	17.2	18.9	1.099	41	1.25	7.29	0.0513	14	3.2
5	129S1/SvImJ	f	66	112	19.7	21.3	1.081	129	1.14	7.22	0.0501	16.3	5.2
10	129S1/SvImJ	m	66	112	24.3	24.7	1.016	119	1.13	7.24	0.0533	17.6	6.8
364	129S1/SvImJ	m	72	114	25.3	27.2	1.075	64	1.25	7.27	0.0596	19.3	5.8
365	129S1/SvImJ	m	72	115	21.4	23.9	1.117	48	1.25	7.28	0.0563	17.4	5.7
366	129S1/SvImJ	m	72	118	24.5	26.3	1.073	59	1.25	7.26	0.0609	17.8	7.1
367	129S1/SvImJ	m	72	122	24	26	1.083	69	1.29	7.26	0.0584	19.2	4.6
6	129S1/SvImJ	m	66	116	21.6	23.3	1.079	78	1.15	7.27	0.0497	17.2	5.7
7	129S1/SvImJ	m	66	107	22.7	26.5	1.167	90	1.18	7.28	0.0493	18.7	7
8	129S1/SvImJ	m	66	108	25.4	27.4	1.079	35	1.24	7.26	0.0538	18.9	7.1
9	129S1/SvImJ	m	66	109	24.4	27.5	1.127	43	1.29	7.29	0.0539	19.5	7.1

Example: Making a Random Sampling

mice.xls

790 mice from different strains

http://phenome.jax.org

ID	Strain	Sex	Starting age	Ending age	Starting weight	Ending weight	Weight change	Bleeding time	Ionized Ca in blood	Blood pH	Bone mineral density	Lean tissues weight	Fat weight
1	129S1/SvImJ	f	66	116	19.3	20.5	1.062	64	1.2	7.24	0.0605	14.5	4.4
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3	129S1/SvImJ	f	66	108	17.9	19.8	1.106	90	1.16	7.26	0.0546	13.8	2.9
368	129S1/SvImJ	f	72	114	18.3	21	1.148	65	1.26	7.22	0.0599	15.4	4.2
369	129S1/SvImJ	f	72	115	20.2	21.9	1.084	55	1.23	7.3	0.0623	15.6	4.3
370	129S1/SvImJ	f	72	116	18.8	22.1	1.176		1.21	7.28	0.0626	16.4	4.3
371	129S1/SvImJ	f	72	119	19.4	21.3	1.098	49	1.24	7.24	0.0632	16.6	5.4
372	129S1/SvImJ	f	72	122	18.3	20.1	1.098	73	1.17	7.19	0.0592	16	4.1
4	129S1/SvImJ	f	66	109	17.2	18.9	1.099	41	1.25	7.29	0.0513	14	3.2
5	129S1/SvImJ	f	66	112	19.7	21.3	1.081	129	1.14	7.22	0.0501	16.3	5.2
10	129S1/SvImJ	m	66	112	24.3	24.7	1.016	119	1.13	7.24	0.0533	17.6	6.8
364	129S1/SvImJ	m	72	114	25.3	27.2	1.075	64	1.25	7.27	0.0596	19.3	5.8
365	129S1/SvImJ	m	72	115	21.4	23.9	1.117	48	1.25	7.28	0.0563	17.4	5.7
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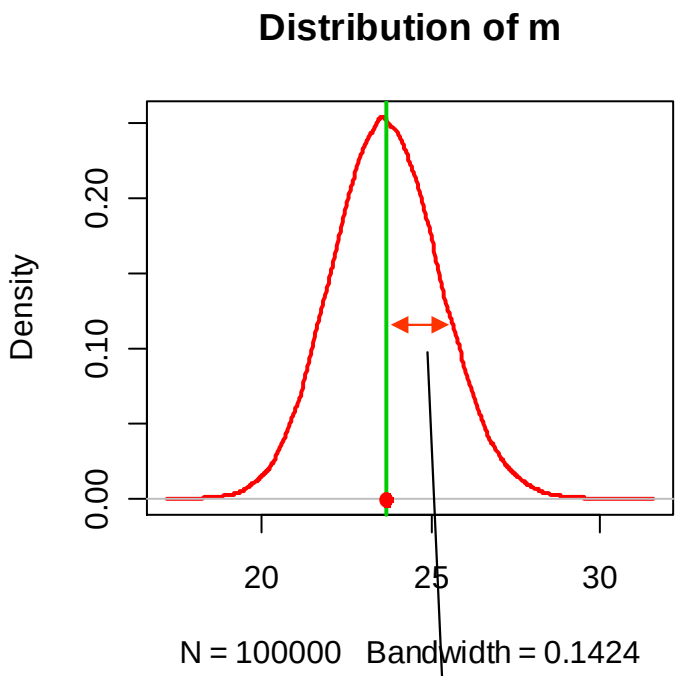
6	129S1/SvImJ	m	66	116	21.6	23.3	1.079	78	1.15	7.27	0.0497	17.2	5.7
7	129S1/SvImJ	m	66	107	22.7	26.5	1.167	90	1.18	7.28	0.0493	18.7	7
8	129S1/SvImJ	m	66	108	25.4	27.4	1.079	35	1.24	7.26	0.0538	18.9	7.1
9	129S1/SvImJ	m	66	109	24.4	27.5	1.127	43	1.29	7.29	0.0539	19.5	7.1

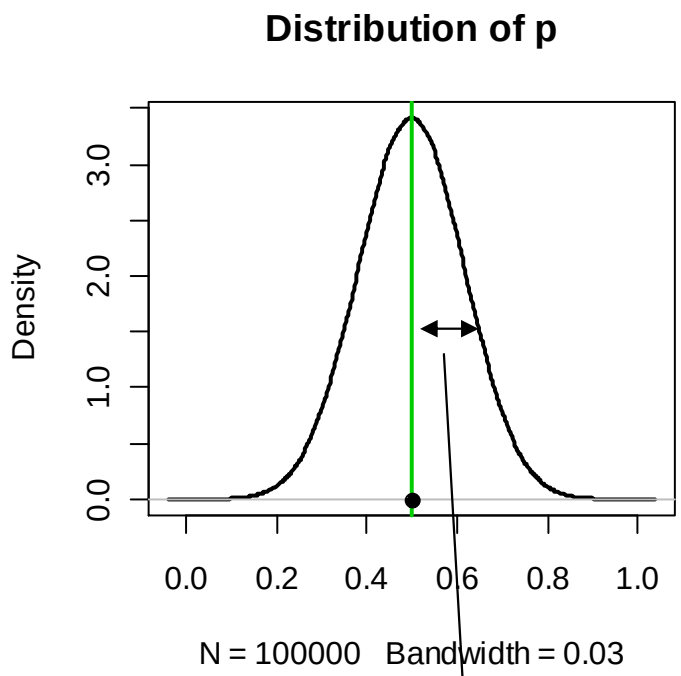
1. Add a column to the table
2. Fill it with **`=RAND()`**
3. Sort all the table by this column
4. Assume that these mice is a population with size N=790. Build 3 samples with *n*=20
5. Calculate *m*, *s* for ending weight and *p* – proportion of males for each sample

Point estimator

The sample statistic, such as *m*, *s*, or *p*, that provides the point estimation the population parameters μ, σ, π.

Sampling distribution
A probability distribution consisting of all possible values of a sample statistic.





$$E(m) = \mu$$

$$E(p) = \pi$$

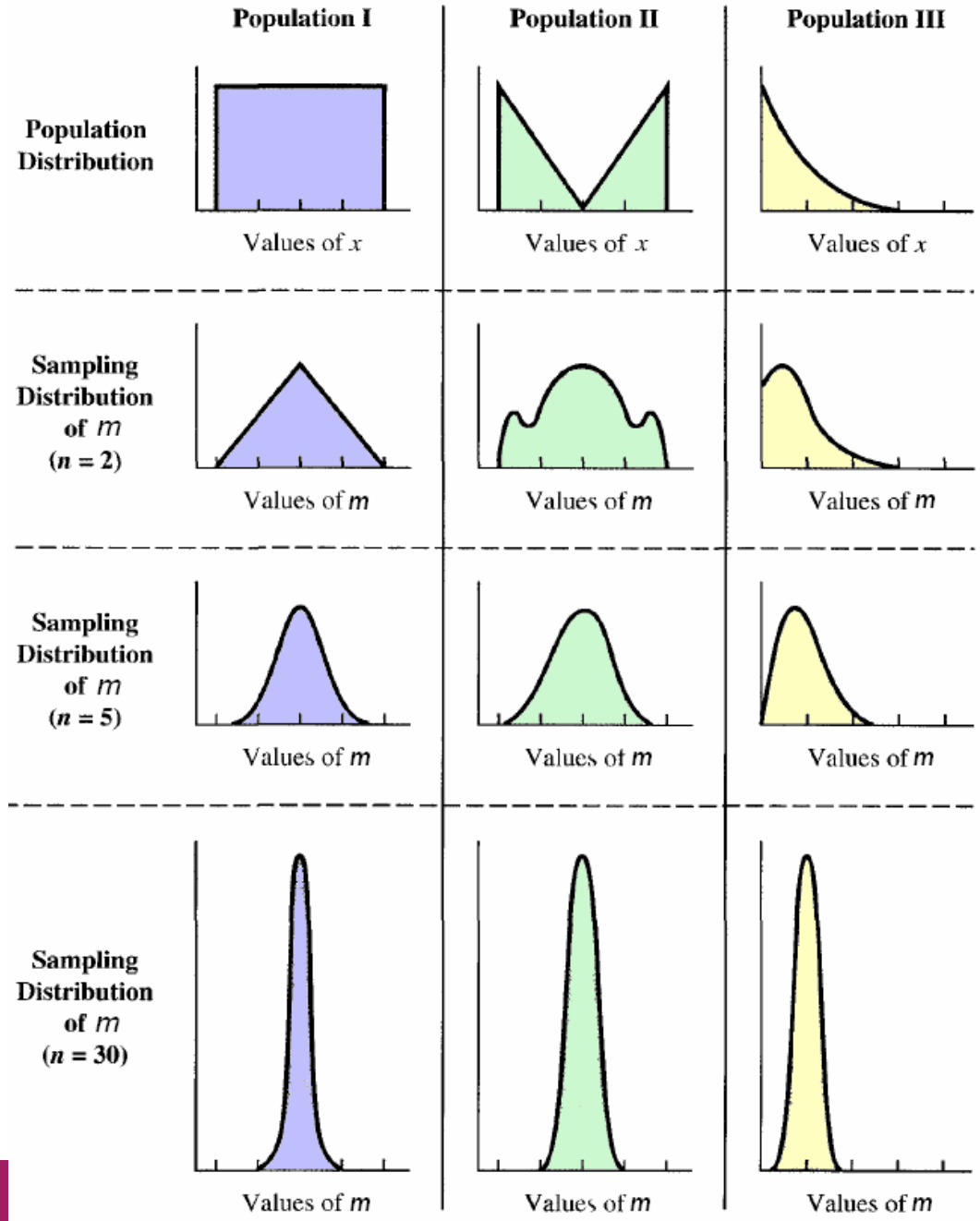
$$\sigma_m = \frac{\sigma}{\sqrt{n}}$$

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}}$$

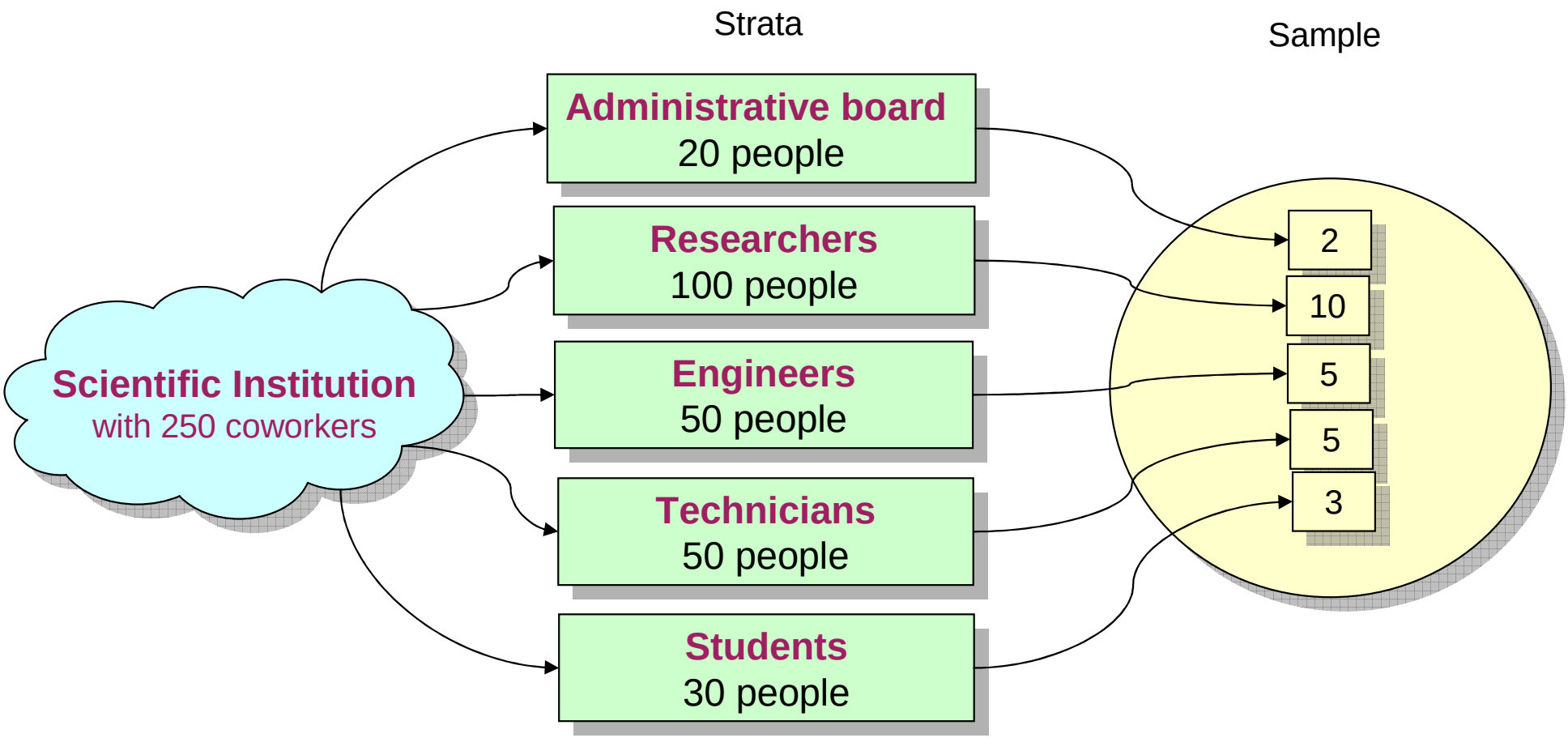
Central limit theorem

In selecting simple random sample of size *n* from a population, the ***sampling distribution of the sample mean m can be approximated by a normal distribution*** as the sample size becomes large

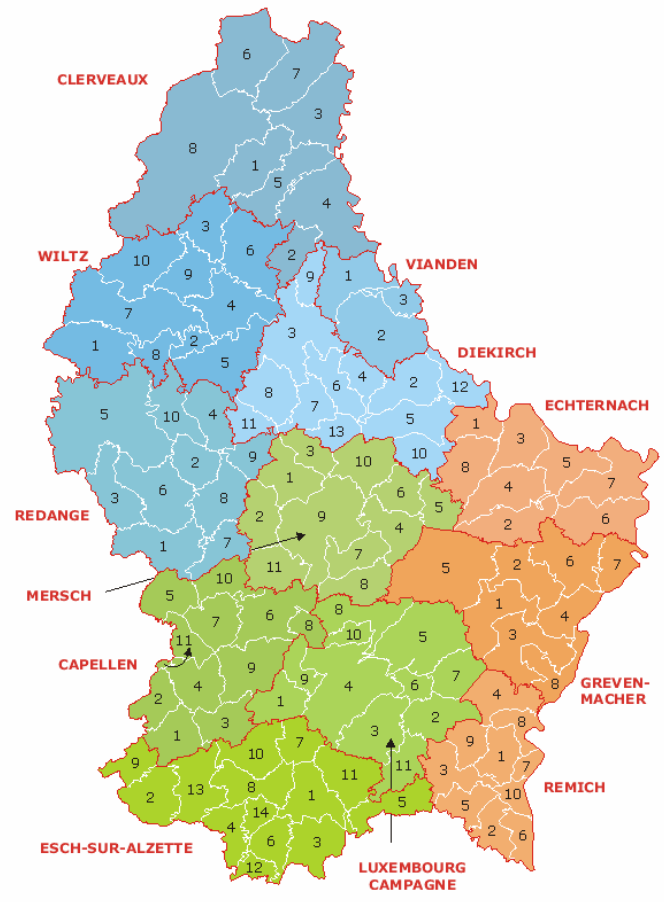
In practice if the sample size is ***n*>30**, the normal distribution is a good approximation for the sample mean for any initial distribution.



Stratified random sampling
A probability sampling method in which the population is first divided into strata and a simple random sample is then taken from each stratum.



Cluster sampling
A probability sampling method in which the population is first divided into clusters and then a simple random sample of the clusters is taken.

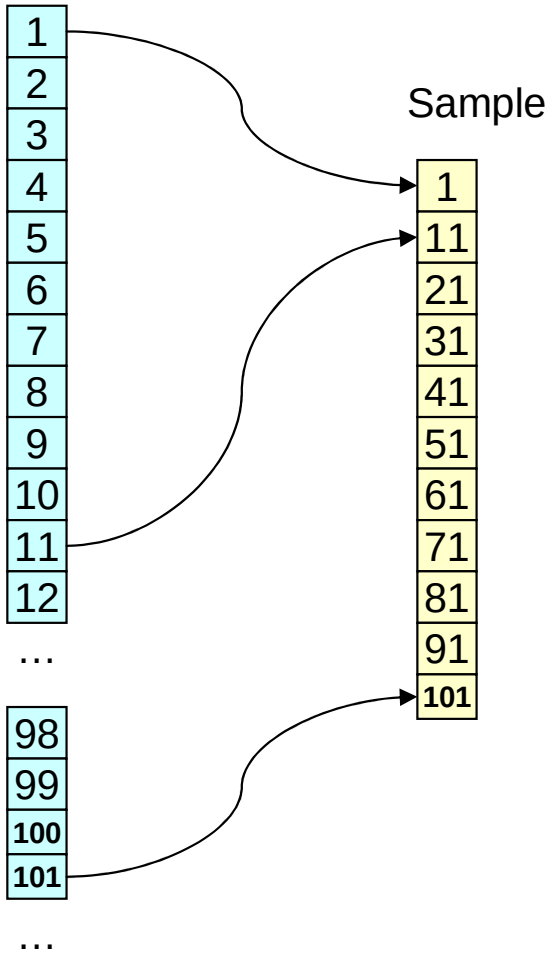


Clusters

- Luxembourg-camp
- Esch-sur-Alzette
- Remich
- Diekirch
- Mersch
- Redange
- … etc.

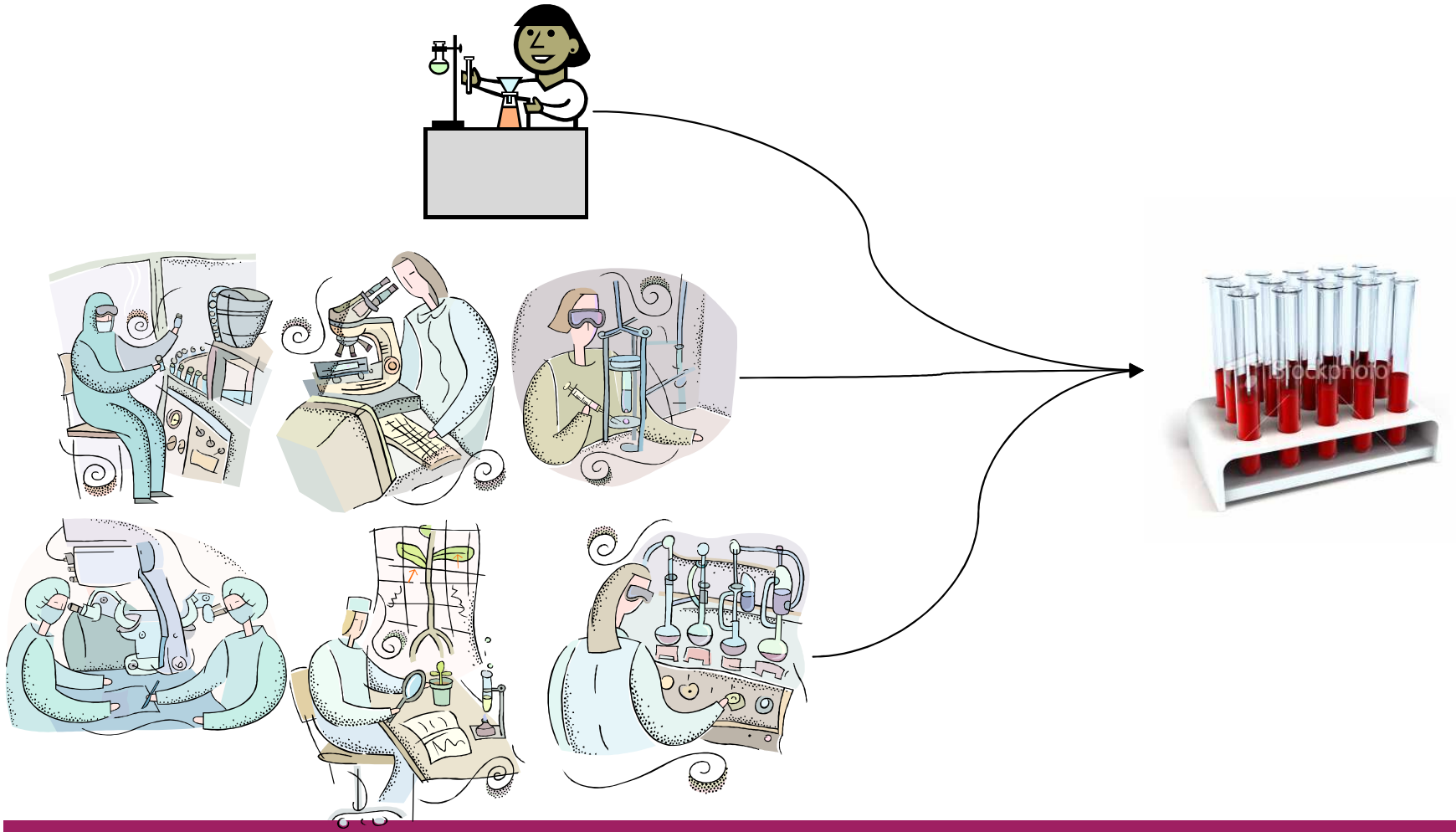
→ **Sample**

Systematic sampling
A probability sampling method in which we randomly select one of the first *k* elements and then select every *k*-th element thereafter.



Convenience sampling

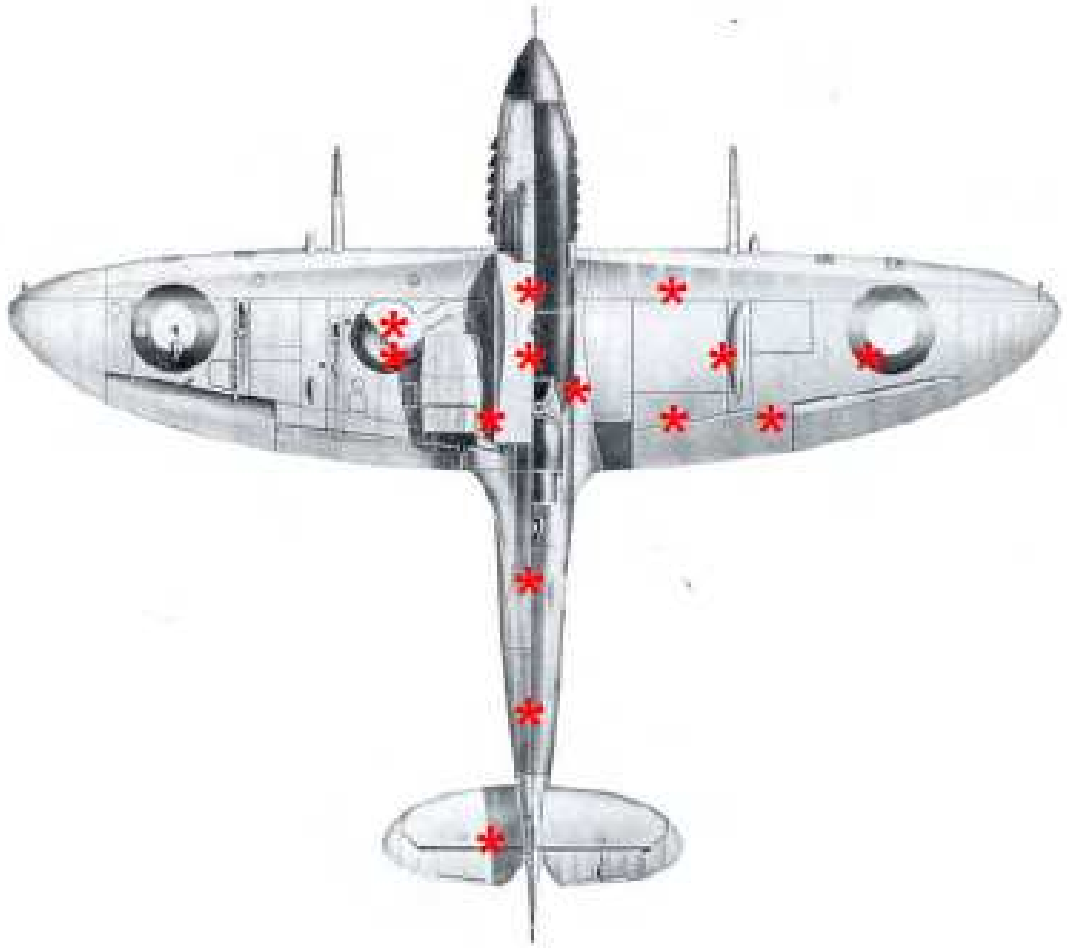
A nonprobability method of sampling whereby elements are selected for the sample on the basis of convenience.



Judgment sampling
A nonprobability method of sampling whereby elements are selected for the sample based on the judgment of the person doing the study.

Perform of a selection of most confident or most experienced experts.

Spitfire: analysis of the damage



Were to put additional protection?

- **Interval estimation**
  - interval estimation
  - population mean: σ known
  - population proportion
  - population mean: σ unknown
  - Student's distribution
  - estimation the size of a sample

POPULATION AND SAMPLE

Parameters

Population parameter
A numerical value used as a summary measure for a population (e.g., the mean μ, variance σ^2, standard deviation σ, *proportion* π)

POPULATION

μ – mean
σ^2 – variance
N – number of elements (usually $N=\infty$)

SAMPLE

$m, \bar{x}$ – mean
s^2 – variance
n – number of elements

Sample statistic
A numerical value used as a summary measure for a sample (e.g., the sample mean m, the sample variance s^2, and the sample standard deviation s)

All existing laboratory
Mus musculus

mice.txt

790 mice from different strains

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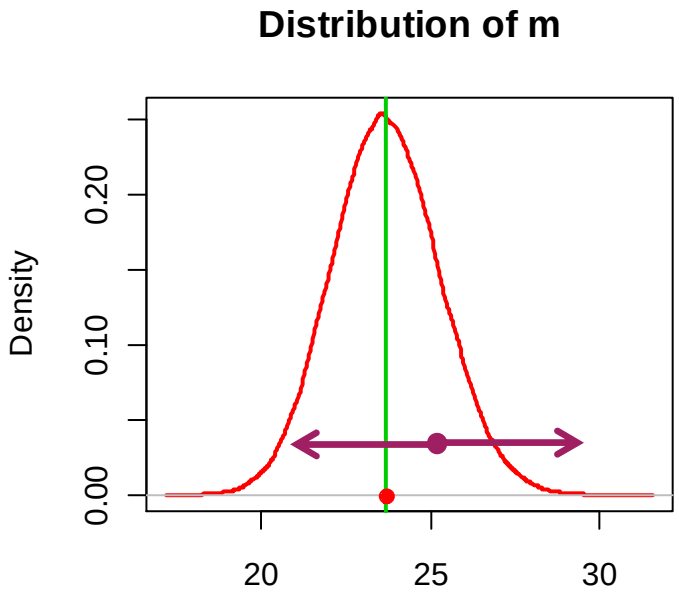
7	129S1/SvImJ	m	66	107	22.7	26.5	1.167	90	1.18	7.28	0.0493	18.7	7
8	129S1/SvImJ	m	66	108	25.4	27.4	1.079	35	1.24	7.26	0.0538	18.9	7.1
9	129S1/SvImJ	m	66	109	24.4	27.5	1.127	43	1.29	7.29	0.0539	19.5	7.1

Interval estimate
An estimate of a population parameter that provides an interval believed to contain the value of the parameter. For the interval estimates in this chapter, it has the form: point estimate ± margin of error.

Margin of error
The ± value added to and subtracted from a point estimate in order to develop an interval estimate of a population parameter.

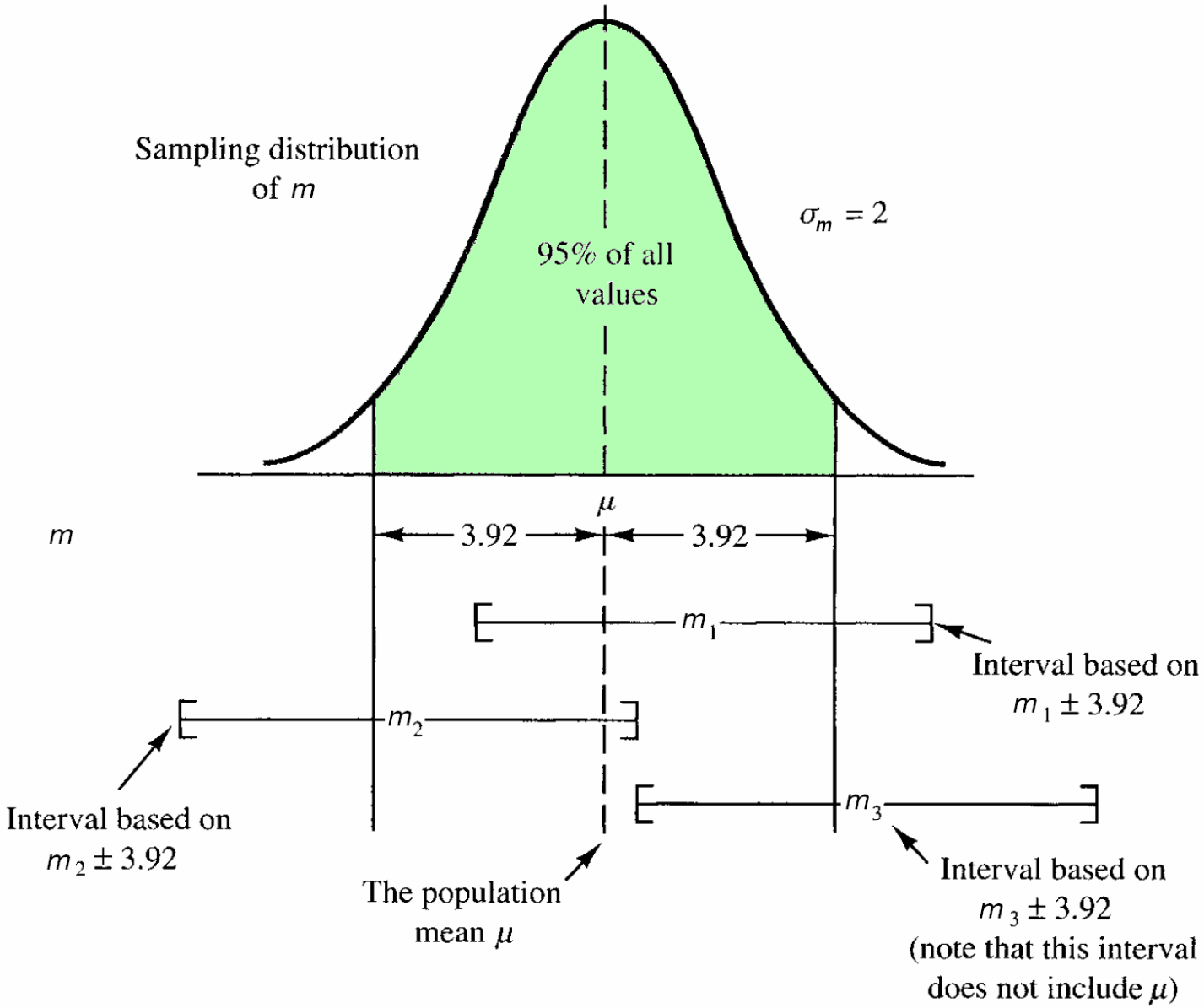


$$\mu = m \pm merror$$

σ known
The condition existing when historical data or other information provides a good value for the population standard deviation prior to taking a sample. The interval estimation procedure uses this known value of o in computing the margin of error.

σ unknown
The condition existing when no good basis exists for estimating the population standard deviation prior to taking the sample. The interval estimation procedure uses the sample standard deviation s in computing the margin of error.

Population Mean: σ Known



Confidence level

The confidence associated with an interval estimate. For example, if an interval estimation procedure provides intervals such that 95% of the intervals formed using the procedure will include the population parameter, the interval estimate is said to be constructed at the 95% confidence level.

Confidence interval

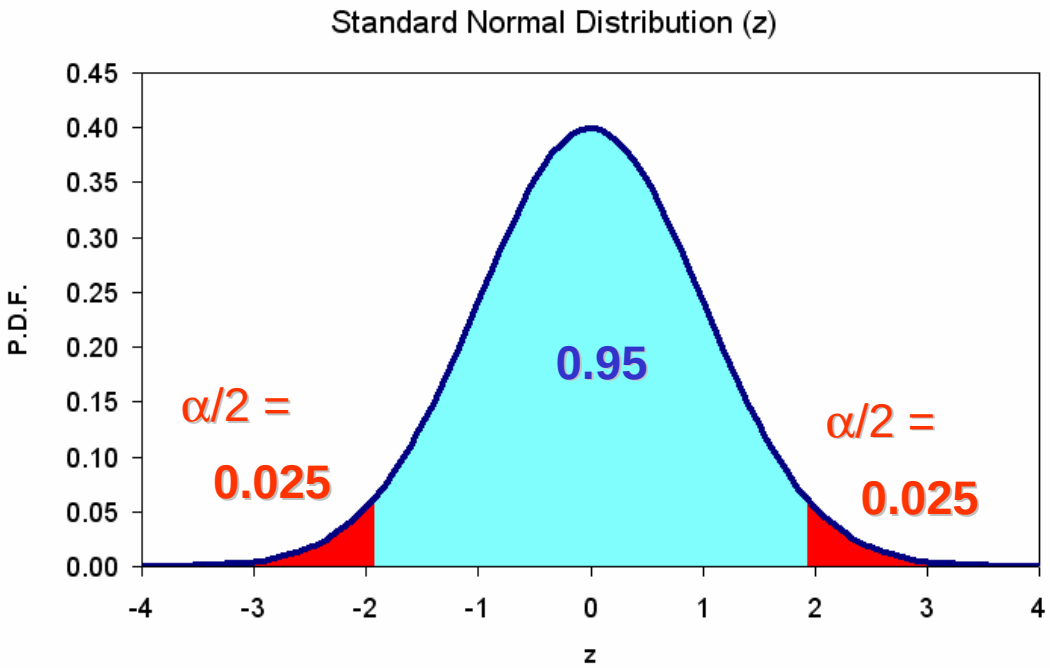
Another name for an interval estimate.

$$\mu = m \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

For 95 % confidence α = 0.05, which means that in each tail we have 0.025. Corresponding $z_{\alpha/2}$ = 1.96

In Excel use one of the following functions:

- `= CONFIDENCE(alpha, σ, n)`
- `= -NORMINV(alpha/2,0,1)*σ/SQRT(n)`

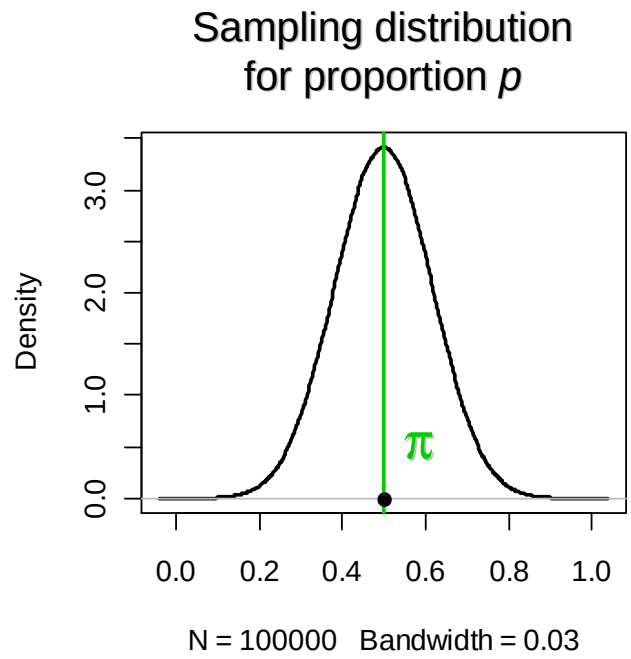


Population Proportion

$$\sigma_p = \sqrt{\frac{\pi(1-\pi)}{n}} \longrightarrow \sigma_p = \sqrt{\frac{p(1-p)}{n}}$$

$$\pi = p \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$

if $np \geq 5$ and $n(1-p) \geq 5$



Practical Work

pancreatitis.txt

Define a 95% confidence interval for **never-smoking** proportion of people coming to a hospital

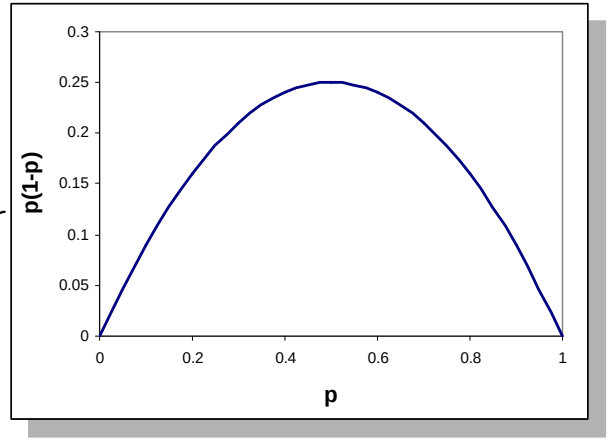
n=	270
p(never)=	**0.214815**
sp=	0.024994
E=	**0.048988**

for 95% confidence $z_{0.025} = 1.96$

$\pi = 21.5 \pm 4.9$ %

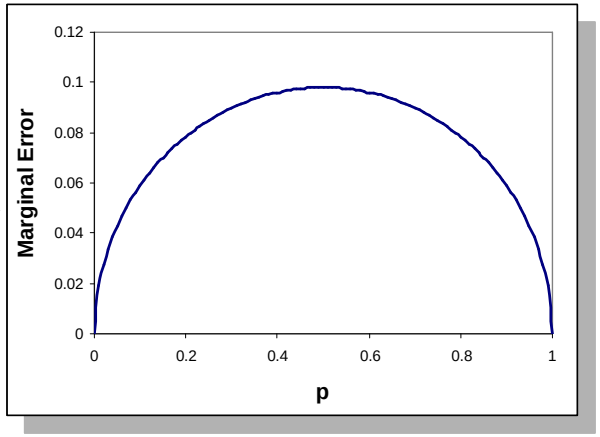
Population Proportion: Some Practical Aspects

$$\pi = p \pm z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}}$$



1. The normal distribution is applicable only when enough data points are observed. The rule of thumb is: $np \geq 5$ and $n(1-p) \geq 5$

2. The maximal marginal error is observed when p=0.5



3. The estimation of the sample size can be obtained:

$$n = \frac{z_{\alpha/2}^2 p(1-p)}{E^2}$$

$np \geq 5$ and $n(1-p) \geq 5$

where p is a best guess for π or the result of a preliminary study

Population Mean: σ Unknown

Assume that we have a sample of 20 mice and would like to estimate an average size of a mice in population.

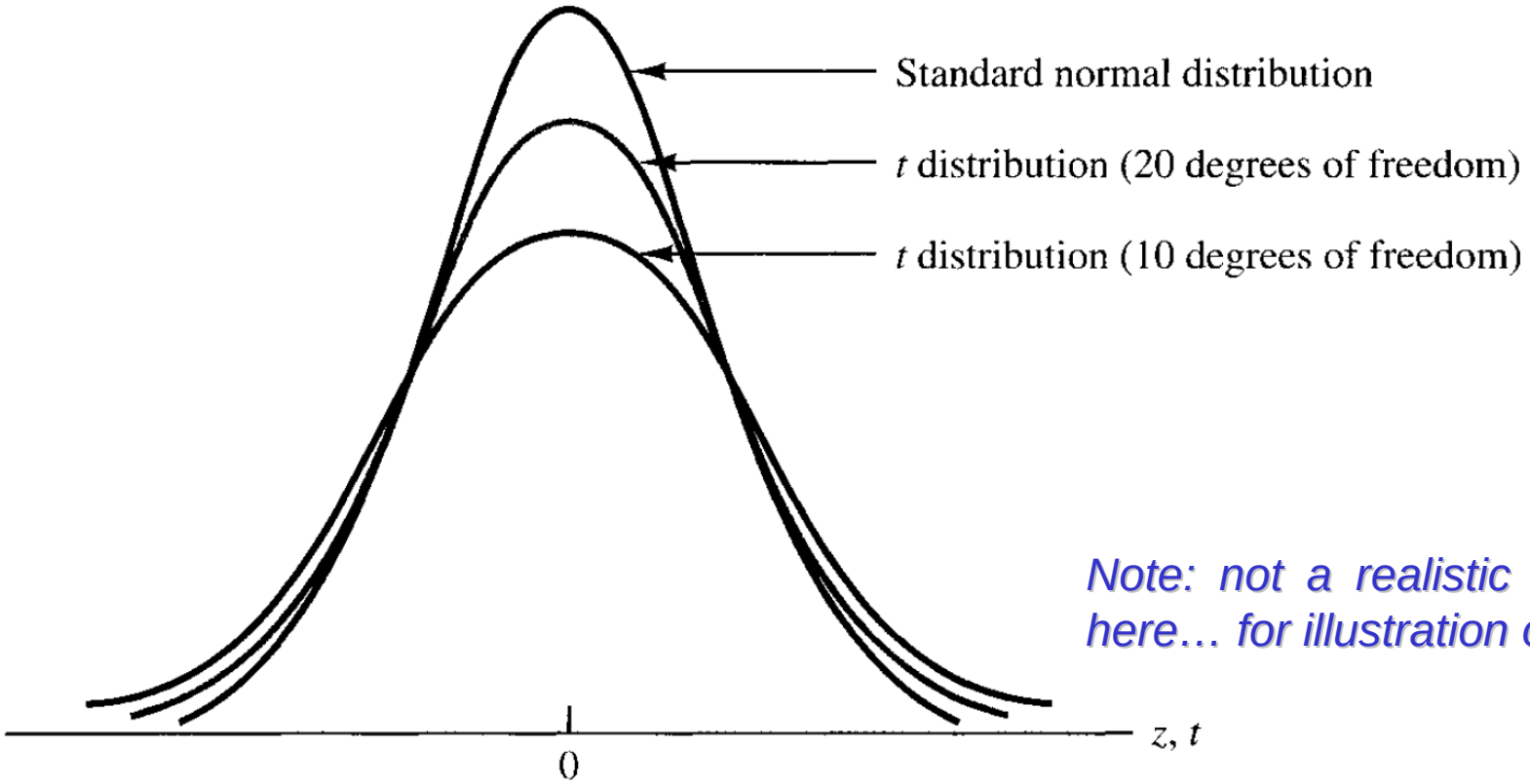
Weight
39.9
19.8
32.4
21
27.5
20.8
21.3
40
10.7
22.6
27
10.8
20.9
14.7
31.4
17.2
11.4
19.1
31.3
14.8

$m = 22.73$

$s = 8.84$

$$\sigma_m = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}$$

As we replace σ → s, we introduce an additional error and this change the distribution from *z* to *t* (Student)



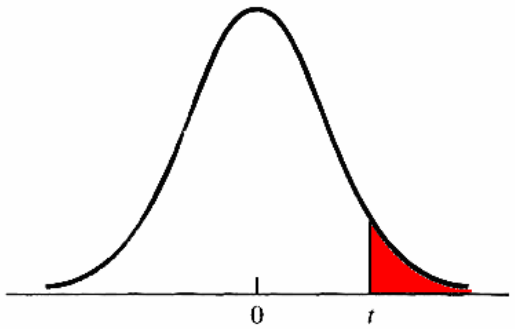
Note: not a realistic scale here… for illustration only

t-distribution

A family of probability distributions that can be used to develop an interval estimate of a population mean whenever the population standard deviation σ is unknown and is estimated by the sample standard deviation s.

Degrees of freedom

A parameter of the t-distribution. When the t distribution is used in the computation of an interval estimate of a population mean, the appropriate t distribution has $n - 1$ degrees of freedom, where n is the size of the simple random sample.



Degrees of Freedom	Area in Upper Tail					
	.20	.10	.05	.025	.01	.005
1	1.376	3.078	6.314	12.706	31.821	63.656
2	1.061	1.886	2.920	4.303	6.965	9.925
3	.978	1.638	2.353	3.182	4.541	5.841
4	.941	1.533	2.132	2.776	3.747	4.604

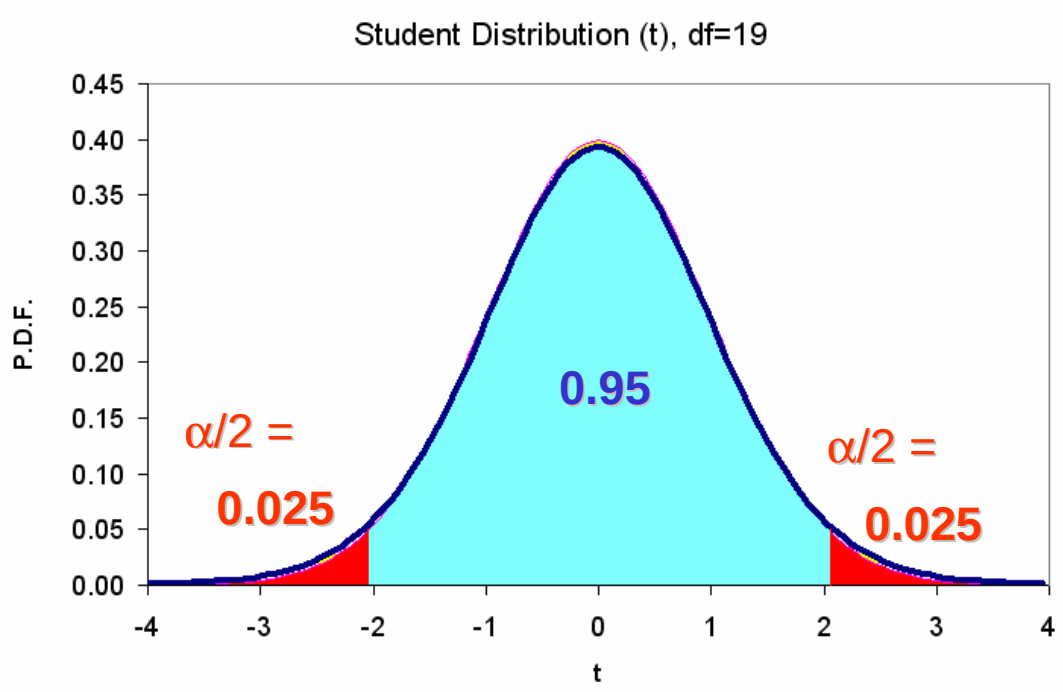
Population Mean: σ Unknown

Weight
39.9
19.8
32.4
21
27.5
20.8
21.3
40
10.7
22.6
27
10.8
20.9
14.7
31.4
17.2
11.4
19.1
31.3
14.8

m =	22.73
s =	8.84

s(m) =	1.98
t =	2.09
m.e. =	4.14

$$\mu = m \pm t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}}$$



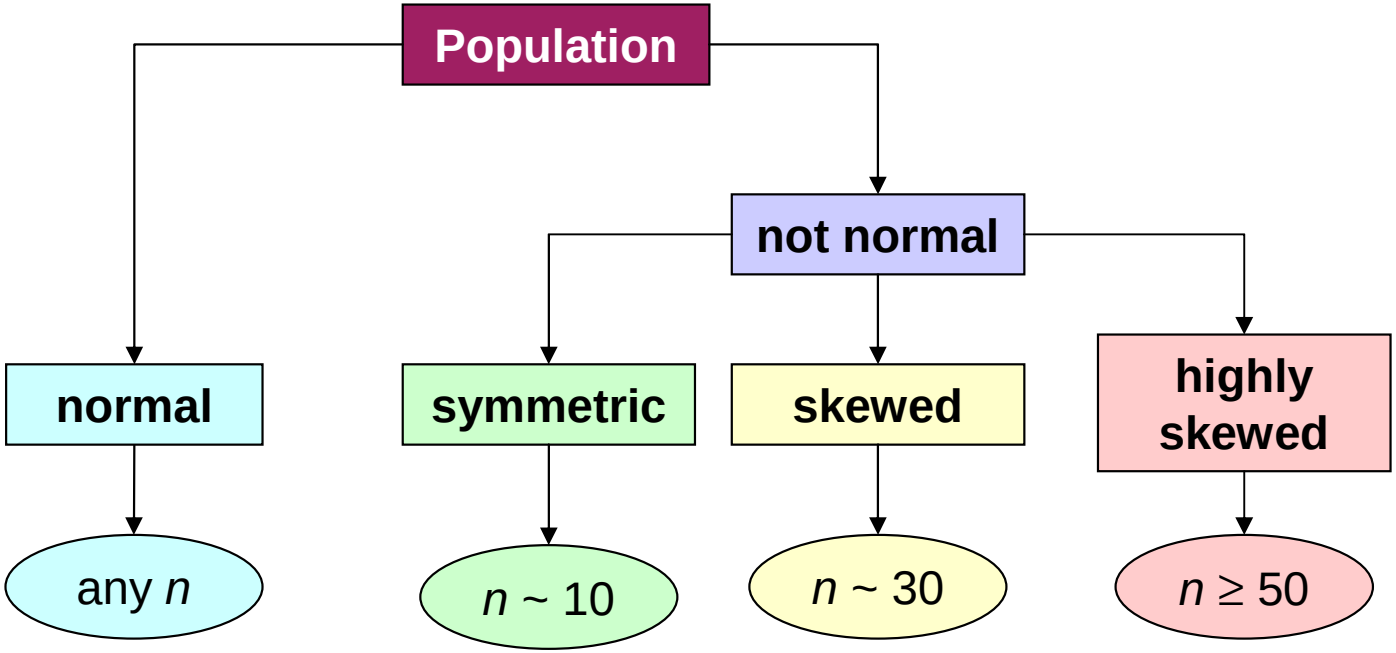
In Excel use:

- `= TINV(alpha, degree-of-freedom)` !!!

Population Mean: Practical Advices

$$\mu = m \pm t_{\alpha/2}^{(n-1)} \frac{s}{\sqrt{n}}$$

Advice 1



Advice 2

if n >100 you can use z-statistics instead of t-statistics (error will be <1.5%)

Let's focus on another aspect: how to select a proper number of experiments.

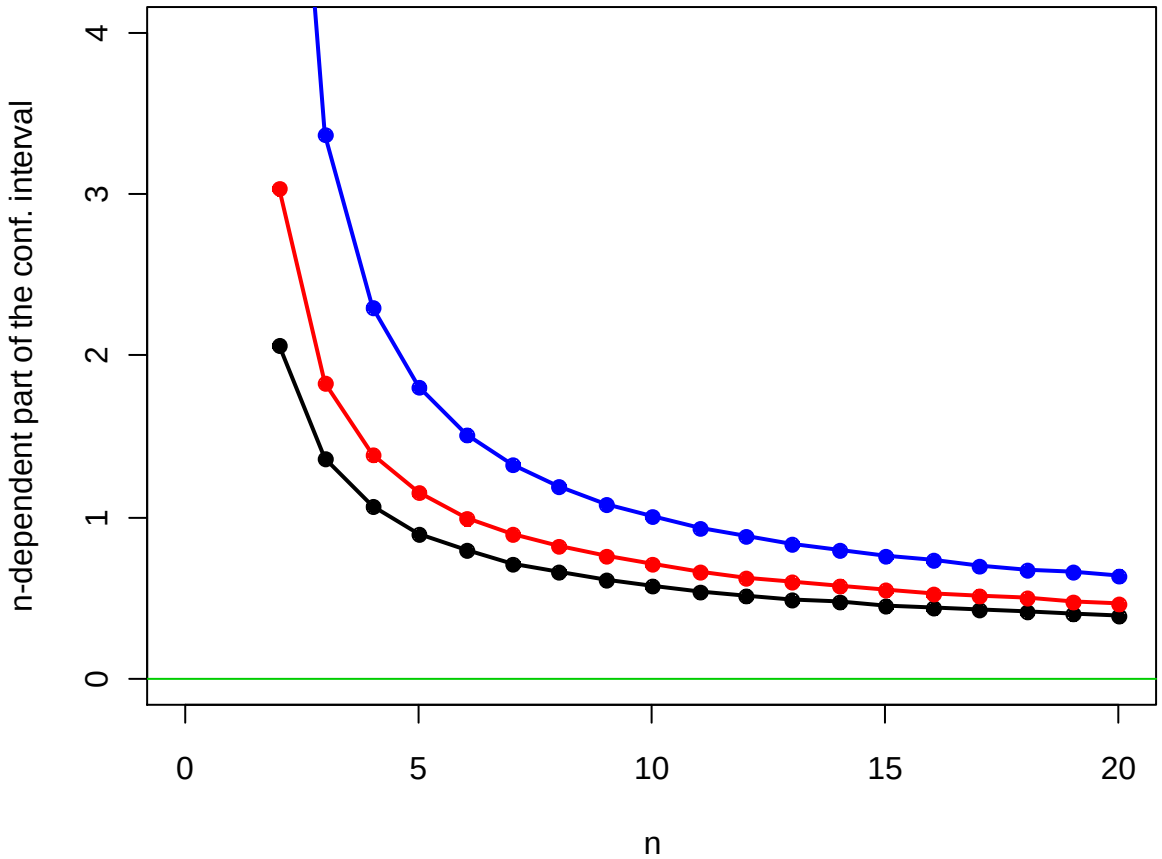
$$\mu = m \pm E(n,\sigma)$$
$$E(n,\sigma) = E$$
$$n - ?$$

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$n = \frac{z_{\alpha/2}^2\sigma^2}{E^2}$$

$$n = \frac{z_{\alpha/2}^2\sigma^2}{E^2}$$

Effect of the Sample Size



Thank you for your attention

to be continued…